STUDY OF CONSTITUTIVE MODELS FOR CYCLIC LIQUEFACTION IN SAND

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ABSTRACT
Liquefaction induced by cyclic loading in the saturated loose granular deposit often proves to be catastrophic. UBCSAND and PM4SAND model were basically developed to meet the need arising for the numerical evaluation of the soil liquefaction. The concept with which each of these models was formulated is essentially different. The UBCSAND model is an effective stress model framed on the basis of classical plasticity theory. The PM4SAND model is developed based on bounding surface plasticity theory embracing the concept of critical state. It considers the bias in fabric orientation during the alternative dilation and contraction. The PM4SAND has recently emerged as a model that accounts for the soil behavior under monotonic and cyclic loading over a wide range of relative density and confining pressure reasonably well. The UBCSAND model has been commonly deployed for analyzing the seismic response of real time geotechnical structures while the PM4Sand model promises its ability in exhibiting the key features observed in laboratory simulations and continues to evolve as well. In this paper the formulation and characteristic response of both the model in predicting the soil behavior under cyclic loading is overviewed and the results of single element simulation of simple shear loading using PM4Sand is presented along with the data of Fraser river sand from the literature to facilitate a comparison of model the responses of UBCSAND and PM4SAND with laboratory observations.

Keywords: UBCSAND PM4SAND Liquefaction

1 Introduction
Soil liquefaction is such a problem whose numerical simulation has received immense attention from the researchers in past couple of decades. The laboratory simulation of soil liquefaction has commenced in late 1960’s which ultimately led to enormous contribution to the earthquake geotechnical engineering field. With the advent of modern computing techniques, the numerical simulation of this problem has begun. The backbone of such technique is to analyze these problems with the competent constitutive models which keeps on evolving from total stress approach to recent complex models that are based on classical elastic- plastic theory and bounding surface plasticity formulations. In this paper the features of state of art constitutive models such as UBCSAND and PM4SAND are elaborated. While the UBCSAND is familiar for its seismic applications across the world, the recently emerged PM4Sand model promises better simulation of the laboratory observations of cyclic simple shear tests.

2 UBCSAND
UBCSAND is an elastic-plastic strain hardening material model devised by Byrne et al.(2004). The material behavior that the model incorporates includes the nonlinearity, stress dependency, induced anisotropy, shear-volume coupling and rotation of principal stress axes. Capturing the aforementioned characteristic behavior with reasonable assumptions enable this model to have reliable insight into the problem of soil liquefaction. Since its development, it has been employed to simulate the numerous static and seismic responses of geotechnical structures. It is an effective stress model that can be operated on a fully coupled basis where the mechanical and groundwater flow calculations can be performed simultaneously. This formulation represents the actual field conditions as some amount of flow is likely to occur during the...
seismic excitation of the ground which can delay the onset of liquefaction to some extent.

The ingredients of the UBCSAND model are described below.

### 2.1 Yield surface

The role of the yield surface in Fig.1 is to delineate the elastic and plastic responses. Once the stress ratio falls outside the yield surface, the plasticity begins. The movement of the yield surface is governed by the hardening rule. The yield surface can expand (or contract) and translate in the stress space that are referred as isotropic and kinematic hardening respectively. Kinematic hardening is employed in this model.

![Fig. 1 Yield locus (after Byrne et al. 2011)](image)

### 2.2 Plastic flow

The non-associated flow rule is adopted in this model. It leads the direction of plastic strains to depend on the slope of the plastic potential surface (and not the yield surface). The evolution of plastic strains with the increase in stress ratio is governed by the hyperbolic stress strain relation (Fig.2) that enforces non-linearity in the model. Dilatancy couples the shear and volumetric response of the soil skeleton. The magnitude of volumetric strains is assessed as a function of dilation angle. The constant volume friction angle parameter separates the contractive and dilative regime.

![Fig. 2 Hyperbolic stress strain in UBCSAND (after Byrne et al. 2011)](image)

### 2.3 Discussion on UBCSAND model

This model is capable of predicting the critical aspects of soil behavior during complex loading. It produces large deformation during the simulation, only when the pore pressure ratio ($r_u$) exceeds 0.5. The model predicts the incremental pore pressures but the significant shear strains that occur during static shearing is not reflected until the flow liquefaction is triggered. The response obtained from the model does not highlight the effect of sustained static shear stresses (Shiriro & Bray 2013). The UBCSAND model exhibits repeating lock-up in stress strain loop.

### 3 PM4Sand model

PM4Sand model evolved as an extended version of DM04 sand model (Dafalias & Manzari 2004) with rigorous modifications introduced in version 2 of the model. The behavior of soil is confined to bounding surface plasticity along with dilatancy and critical state surface to represent the response of soil under seismic loading over a range of density and confining pressure with this unique formulation of the model. The critical state surface is positioned based on the relative state parameter index $\xi_R$ which is consistent Bolton’s (1986) relative dilatancy index relationship. The other two surfaces such as virtual bounding and virtual dilatancy surfaces (depicted in Fig.3 and Fig.4) are placed relative to the critical state surface based on the parameters $n_b$ and $n_d$ that are calibrated to produce the soil response consistent with expected behavior. These two surfaces are not real since their positions continue to vary with respect to $\xi_R$ until the current stress-ratio meets these surfaces at different values of it. $\xi_R$ is a measure of in-situ state of sand since it varies with respect to relative density and confining stress. This parameter which makes the model critical state compatible acts as a means to represent the soil behavior over a wide range of in-situ conditions. The effect of overburden and static shear stresses are addressed through this parameter which makes the model elite.

#### 3.1 Yield surface

The wedge shape yield surface is adopted in this model. The stress state in the soil is tracked by image back stress ratio (since there is a image bounding and dilatancy stress ratio) to enable easy kinematic hardening process.

#### 3.2 Bounding surface

It reflects the peak strength of the soil and it marks the onset of softening response and progressive shearing of soil towards critical state. Usually dense sands on shearing reach the bounding surface to show the distinct peak in hardening and immediately followed by softening. In the case of loose sand, the bounding surface lies on or close to critical state so that it can exhibit hardening until failure without any dilation.
3.3 Dilatancy surface

It separates the contractive behavior from dilative volumetric behavior. This surface lies below the critical surface in dense soil conditions where the dilation leads to the peak behavior. But the loose sand does not dilate and hence dilatancy surface remains close to the critical state surface.

Fig. 3 Loading surfaces in PM4Sand version 3 (after Boulanger & Ziotopoulou 2015)

3.4 Hardening of soil

The soil modulus varies with the distance between the current stress state and the corresponding image back stress ratio $\alpha$ on bounding surface as seen in Fig.4. Hence the non-linear variation of the hardening modulus is ensured. The stress reversal is identified with reference to the current, initial and previous initial back stress ratios. The over stiffening of modulus during partial loading - unloading cycles is averted.

3.5 Fabric parameter

The fabric parameter is introduced in DM04 model to enhance the contractiveness of sand by accounting the changes in fabric orientation that occurs in dilative phase of deformation. This dilation induced fabric changes cause drastic increase in pore pressures once the loading reverts to compression after reversal. The experimental evidence supports the changes observed in particle contact-normal orientation distribution of fabric during dilation phase (Nemat-Nasser 1981). The plastic modulus and dilatancy relationship is made to depend on fabric parameter $Z$. The fabric variable posses negative and positive value during the dilation in compression and extension loading respectively. The contractive tendency of the soil is enhanced only when the fabric is favorable ($Z:n>0$). The shortcomings in the DM04 model include i) the effect of fabric parameter vanishes after a number of loading cycles. ii) the secant shear modulus remains non -degraded and it results in alternating plasticity behavior (Vytiniotis 2012). The actual fabric parameter in the original DM04 model itself is regarded as a cause of the problem. So in PM4Sand model the fabric parameter is made to evolve continuously and the secant shear modulus is made to rely on fabric parameter in a bid to overcome these shortcomings.

3.6 Modifications to PM4Sand version 2

Adopting the Version 2 of PM4Sand model to simulate the effect on sloping ground subjected to irregular cyclic loading resulted in simulated response which is inconsistent with the laboratory observations. The stress reversal has a pronounced effect on the onset of dilation at a particular stress ratio. The dilation begins at relatively smaller stress ratio in the non-reversal case when compared to that of the reversal loading. To account this a rotated dilatancy surface is introduced in PM4Sand model version3 (Boulanger & Ziotopoulou 2015) to impart early dilation in loading paths which does not encounter the stress reversal. The unrealistic softening of plastic modulus due to the discrepancies in tracking the back-stress ratios during the partial loading cycles (without stress reversal) is sorted out. This leads to improved capability of the model which comprehensively accounts for the effects of state of sand and loading history. The emphasis was given to calibrate the model to target values of CRR (from liquefaction triggering chart). The three primary input parameters include relative density $I_D$, elastic shear modulus co-efficient $G_0$ and contraction rate parameter $h_{0p}$ (to calibrate the model for specific CSR target values). The first two parameters can be availed easily through correlations with SPT ‘N’, in-situ shear wave velocity which are often used in practice. The secondary input parameters were assigned with default value arrived based on the generalized calibration process (Boulanger & Ziotopoulou 2015).

The PM4Sand model produces a gradual accumulation of shear strains as depicted in Fig.5 under cyclic loading. The fabric and tracking of its history is attributed for these developments in prediction capability of the model. The functional form of
constitutive relationships adopted is believed to bolster the model. The input parameters such as $D_r$, $h_{p0}$, $G_0$ and $Z_{\text{max}}$ are derived from calibration as 0.39, 0.28, 512 and 20 respectively for the results presented below. Parameters other than the above mentioned, are assigned with default values (Boulanger, R. W & Ziotopoulou, K. 2015). Figs. 5 & 6 illustrates the results of single element simulation of Cyclic Direct Simple Shear (CDSS) loading using UBCSAND and PM4Sand model in FLAC 2D 7.0. The reasonable agreement between the simulated single element response in UBCSAND & CDSS test with PM4SAND model simulation is achieved. The analysis is carried out in Fraser river sand ($N_1=6.9$) that had liquefied in 17 number of loading cycles with an applied CSR of 0.0826 under a initial vertical effective stress of 100 kPa as observed in CDSS test.

**CONCLUSION**

The principle of operation behind the UBCSAND and PM4Sand model formulations are explored in this paper. They are designed to accommodate the empirical trends established from the laboratory and field investigations. The differences in prediction of soil behavior using both the models emphasizes the significance of fabric dilatancy tensor, the means of representing the state of soil in numerical model and the ease of calibration. These entities in the PM4Sand model puts it in command for evaluation of liquefaction problems over the UBCSAND model. PM4SAND can be comfortably deployed with simple calibration process since the input parameters are designed such that they can be correlated with commonly available in-situ tests.

**References**

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