# METHODS OF ANALYSIS OF PILED RAFT FOUNDATIONS

## A Report Prepared on Behalf of Technical Committee TC18 on Piled Foundations

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**International Society of Soil Mechanics and Geotechnical Engineering**

**July 2001**

# METHODS OF ANALYSIS OF PILED RAFT FOUNDATIONS

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## ABSTRACT

Piled raft foundations provide an economical foundation option for circumstances where the performance of the raft alone does not satisfy the design requirements. Under these situations, the addition of a limited number of piles may improve the ultimate load capacity, the settlement and differential settlement performance, and the required thickness of the raft.

This report summarizes the philosophy of using piles as settlement reducers, and outlines the key requirements of design methods for rafts enhanced with piles. A number of available methods of analysis of piled raft behaviour are reviewed, and their capabilities and limitations are discussed. Some of the methods are useful only for preliminary design or for checking purposes, while others are capable of giving detailed performance predictions and can be used for detailed design. Conclusions are reached regarding the utility of some of the current methods used for design and the limitations of two-dimensional numerical analyses.

A summary is also given of some recent research on the analysis of piled rafts subjected to lateral loadings.

## 1. INTRODUCTION

In the past few years, there has been an increasing recognition that the use of piles to reduce raft settlements and differential settlements can lead to considerable economy without compromising the safety and performance of the foundation. Such a foundation makes use of both the raft and the piles, and is referred to here as a pile-enhanced raft or a piled raft. Technical CommitteeTC18 of the International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE) has focussed its efforts since 1994 towards piled raft foundations, and has collected considerable information on case histories and methods of analysis and design. Comprehensive reports on these activities have been produced by O'Neill at al (2001) and by van Impe and Lungu (1996). In addition, an independent treatise on numerical modelling of piled rafts has been presented by El-Mossallamy and Franke, 1997). Despite this recent activity, the concept of piled raft foundations is by no means new, and has been described by several authors, including Zeevaert (1957), Davis and Poulos (1972), Hooper (1973), Burland et al (1977), Sommer et al (1985), Price and Wardle (1986), Franke (1991), Hansbo (1993), and Franke et al (1994), among many others. Various methods of analysis of piled raft foundations have also been developed, over the past decade in particular, but there appears to be only limited information on the comparative performance of these methods in predicting foundation behaviour.

This report is an update on the report prepared by van Impe and Lungu (1996), and reviews the general philosophies of piled raft design, and the design issues which need to be addressed. Various methods of analysis of piled raft foundations are then reviewed, and their capabilities and limitations are discussed. The methods are then applied to a simplified problem to allow comparison of the performance predicted by the various methods, and an assessment of problems in employing some of the methods. Finally, some aspects of the analysis of laterally loaded piled rafts are reviewed.

## 2. DESIGN CONCEPTS

### 2.1 Alternative Design Philosophies

Randolph (1994) has defined clearly three different design philosophies with respect to piled rafts:

- The "conventional approach", in which the piles are designed as a group to carry the major part of the load, while making some allowance for the contribution of the raft, primarily to ultimate load capacity.
- "Creep Piling" in which the piles are designed to operate at a working load at which significant creep starts to occur, typically 70-80% of the ultimate load capacity. Sufficient piles are included to reduce the net contact pressure between the raft and the soil to below the preconsolidation pressure of the soil.
- Differential settlement control, in which the piles are located strategically in order to reduce the differential settlements, rather than to substantially reduce the overall average settlement.

In addition, there is a more extreme version of creep piling, in which the full load capacity of the piles is utilized, i.e. some or all of the piles operate at 100% of their ultimate load capacity. This gives rise to the concept of using piles primarily as settlement reducers, while recognizing that they also contribute to increasing the ultimate load capacity of the entire foundation system.

Clearly, the latter  three approaches are most conducive to economical foundation design, and will be given special attention herein. However, it should be emphasized that the analysis and design methods to be discussed allow any of the above design philosophies to be implemented.

De Sanctis et al (2001) and Viggiani (2001) have distinguished between two classes of piled raft foundations:

1. "Small" piled rafts, where the primary reason for adding the piles is to increase the factor of safety (this typically involves rafts with widths between 5 and 15 m);
2. "Large" piled rafts, whose bearing capacity is sufficient to carry the applied load with a reasonable safety margin, but piles are required to reduce settlement or differential settlement. In such cases, the width of the raft is large in comparison with the length of the piles (typically, the width of the piles exceeds the length of the piles).

These two categories broadly mirror the conventional and creep piling philosophies considered by Randolph.

Figure 1 illustrates, conceptually, the load-settlement behaviour of piled rafts designed according to the first two strategies. Curve O shows the behaviour of the raft alone, which in this case settles excessively at the design load. Curve 1 represents the conventional design philosophy, for which the behaviour of the pile-raft system is governed by the pile group behaviour, and which may be largely linear at the design load. In this case, the piles take the great majority of the load. Curve 2 represents the case of creep piling where the piles operate at a lower factor of safety, but because there are fewer piles, the raft carries more load than for Curve 1. Curve 3 illustrates the strategy of using the piles as settlement reducers, and utilizing the full capacity of the piles at the design load. Consequently, the load-settlement may be nonlinear at the design load, but nevertheless, the overall foundation system has an adequate margin of safety, and the settlement criterion is satisfied. Therefore, the design depicted by Curve 3 is acceptable and is likely to be considerably more economical than the designs depicted by Curves 1 and 2.

### 2.2 Design Issues

As with any foundation system, a design of a piled raft foundation requires the consideration of a number of issues, including:

1. Ultimate load capacity for vertical, lateral and moment loadings
2. Maximum settlement
3. Differential settlement
4. Raft moments and shears for the structural design of the raft
5. Pile loads and moments, for the structural design of the piles.

In much of the available literature, emphasis has been placed on the bearing capacity and settlement under vertical loads. While this is a critical aspect, the other issues must also be addressed. In some cases, the pile requirements may be governed by the overturning moments applied by wind loading, rather than the vertical dead and live loads.

## 3. CLASSIFICATION OF METHODS OF ANALYSIS

Several methods of analyzing piled rafts have been developed, and some of these have been summarized by Poulos et al (1997). Three broad classes of analysis method have been identified:

- Simplified calculation methods
- Approximate computer-based methods
- More rigorous computer-based methods.

Simplified methods include those of Poulos and Davis (1980), Randolph (1983,1994), van Impe and Clerq (1995), and Burland (1995). All involve a number of simplifications in relation to the modelling of the soil profile and the loading conditions on the raft.

The approximate computer-based methods include the following broad approaches:

- Methods employing a “strip on springs” approach, in which the raft is represented by a series of strip footings, and the piles are represented by springs of appropriate stiffness (e.g. Poulos, 1991)
- Methods employing a “plate on springs” approach, in which the raft is represented by a plate and the piles as springs (e.g. Clancy and Randolph, 1993; Poulos, 1994; Viggiani, 1998; Anagnastopoulos and Georgiadis, 1998).

The more rigorous methods include:

- Boundary element methods, in which both the raft and the piles within the system are discretized, and use is made of elastic theory (e.g. Butterfield and Banerjee, 1971; Brown and Wiesner, 1975; Kuwabara, 1989; Sinha, 1997)
- Methods combining boundary element for the piles and finite element analysis for the raft (e.g. Hain and Lee, 1978; Ta and Small, 1996; Franke et al, 1994; Russo and Viggiani, 1998)
- Simplified finite element analyses, usually involving the representation of the foundation system as a plane strain problem (Desai,1974) or an axi-symmetric problem (Hooper, 1974), and corresponding finite difference analyses via the commercial program FLAC (e.g. Hewitt and Gue, 1994)

- Three-dimensional finite element analyses (e.g. Zhuang et al, 1991; Lee, 1993; Wang, 1995; Katzenbach et al, 1998) and finite difference analyses via the commercial program FLAC 3D.

As a means of summarizing the capabilities of some of the various methods mentioned above, Table 1 lists the methods and summarizes their ability to predict the response of the foundation system.

In the following section, a more detailed description will be given of a limited number of the above methods, and these will then be used to analyze a relatively simple hypothetical problem.

## 4. SIMPLIFIED ANALYSIS METHODS

### 4.1 Poulos-Davis-Randolph (PDR) Method

For assessing vertical bearing capacity of a piled raft foundation using simple approaches, the ultimate load capacity can generally be taken as the lesser of the following two values:

- The sum of the ultimate capacities of the raft plus all the piles
- The ultimate capacity of a block containing the piles and the raft, plus that of the portion of the raft outside the periphery of the piles.

For estimating the load-settlement behaviour, an approach similar to that described by Poulos and Davis (1980) can be adopted. However, a useful extension to this method can be made by using the simple method of estimating the load sharing between the raft and the piles, as outlined by Randolph (1994). The definition of the pile problem considered by Randolph is shown in Figure 2. Using his approach, the stiffness of the piled raft foundation can be estimated as follows:

$$K_{pr} = (K_p + K_r\,(1-\alpha_{cp})) \,/\, (1- \alpha_{cp}{}^{2}\, K_r \,/\, K_p) \qquad (1)$$

where $K_{pr}$ = stiffness of piled raft
$K_p$ = stiffness of the pile group
$K_r$ = stiffness of the raft alone
$\alpha_{cp}$ = raft – pile interaction factor.

The raft stiffness $K_r$ can be estimated via elastic theory, for example using the solutions of Fraser and Wardle (1976) or Mayne and Poulos (1999). The pile group stiffness can also be estimated from elastic theory, using approaches such as those described by Poulos and Davis (1980), Fleming et al (1992) or Poulos (1989). In the latter cases, the single pile stiffness is computed from elastic theory, and then multiplied by a group stiffness efficiency factor which is estimated approximately from elastic solutions.

The proportion of the total applied load carried by the raft is:

$$P_r \,/\, P_t = K_r\,(1- \alpha_{cp}) \,/\, (K_p + K_r\,(1- \alpha_{cp})) = X \qquad (2)$$

where $P_r$ = load carried by the raft
$P_t$ = total applied load.

The raft – pile interaction factor $a_{cp}$ can be estimated as follows:

$$\alpha_{cp} = 1 - \ln\,(r_c \,/\, r_0) \,/\, \zeta \qquad (3)$$

where $r_c$ = average radius of pile cap, (corresponding to an area equal to the raft area divided by number of piles)
$r_0$ = radius of pile
Φιγυρε 12 = $\ln (r_m / r_0)$
$r_m$ = $\{0.25+\xi [2.5 \rho (1-\nu) – 0.25) * L$
Φιγυρε 12 = $E_{sl} / E_{sb}$
Φιγυρε 12 = $E_{sav} / E_{sl}$
Φιγυρε 12 = Poissons ratio of soil
L = pile length
$E_{sl}$ = soil Young’s modulus at level of pile tip
$E_{sb}$ = soil Young’s modulus of bearing stratum below pile tip
$E_{sav}$ = average soil Young’s modulus along pile shaft.

The above equations can be used to develop a tri-linear load-settlement curve as shown in Figure 3. First, the stiffness of the piled raft is computed from equation (1) for the number of piles being considered. This stiffness will remain operative until the pile capacity is fully mobilized. Making the simplifying assumption that the pile load mobilization occurs simultaneously, the total applied load, $P_1$, at which the pile capacity is reached is given by:

$$P_1 = P_{up} / (1-X) \qquad (4)$$

where $P_{up}$ = ultimate load capacity of the piles in the group
X = proportion of load carried by the piles (Equation 2).

Beyond that point (Point A in Figure 3), the stiffness of the foundation system is that of the raft alone ($K_r$), and this holds until the ultimate load capacity of the piled raft foundation system is reached (Point B in Figure 3). At that stage, the load-settlement relationship becomes horizontal.

The load – settlement curves for a raft with various numbers of piles can be computed with the aid of a computer spreadsheet or a mathematical program such as MATHCAD. In this way, it is simple to compute the relationship between the number of piles and the average settlement of the foundation. Such calculations provide a rapid means of assessing whether the design philosophies for creep piling or full pile capacity utilization are likely to be feasible.

### 4.2 Burland’s Approach

When the piles are designed to act as settlement reducers and to develop their full geotechnical capacity at the design load, Burland (1995) has developed the following simplified process of design:

- Estimate the total long-term load-settlement relationship for the raft without piles (see Figure 4). The design load $P_0$ gives a total settlement $S_0$.
- Assess an acceptable design settlement $S_d$, which should include a margin of safety.
- P1 is the load carried by the raft corresponding to $S_d$.
- The load excess $P_0 – P_1$ is assumed to be carried by settlement-reducing piles. The shaft resistance of these piles will be fully mobilized and therefore no factor of safety is applied. However, Burland suggests that a “mobilization factor” of about 0.9 be applied to the ‘conservative best estimate’ of ultimate shaft capacity, $P_{su}$.
- If the piles are located below columns which carry a load in excess of $P_{su}$, the piled raft may be analyzed as a raft on which reduced column loads act. At such columns, the reduced load $Q_r$ is:

$$Q_r = Q - 0.9\ P_{su} \qquad (5)$$

- The bending moments in the raft can then be obtained by analyzing the piled raft as a raft subjected to the reduced loads $Q_r$.
- The process for estimating the settlement of the piled raft is not explicitly set out by Burland, but it would appear reasonable to adopt the approximate approach of Randolph (1994) in which:

$$S_{pr} = S_r * K_r / K_{pr} \qquad (6)$$

where $S_{pr}$ = settlement of piled raft
$S_r$ = settlement of raft without piles subjected to the total applied loading
$K_r$ = stiffness of raft
$K_{pr}$ = stiffness of piled raft.

Equation 1 can be used to estimate $K_{pr}$.

## 5. APPROXIMATE COMPUTER METHODS

### 5.1 Strip on Springs Approach (GASP)

An example of a method in this category is that presented by Poulos (1991) and illustrated in Figure 5. A section of the raft is represented by a strip, and the supporting piles by springs. Approximate allowance is made for all four components of interaction (raft-raft elements, pile-pile, raft-pile, pile-raft), and the effects of the parts of the raft outside the strip section being analyzed are taken into account by computing the free-field soil settlements due to these parts. These settlements are then incorporated into the analysis, and the strip section is analyzed to obtain the settlements and moments due to the applied loading on that strip section and the soil settlements due to the sections outside the raft.

The method has been implemented via a computer program GASP (Geotechnical Analysis of Strip with Piles) and has been shown to give settlement which are in reasonable agreement with more complete methods of analysis. However, it does have some significant limitations, especially as it cannot consider torsional moments within the raft, and also because it may not give consistent settlements at a point if strips in two directions through that point are analyzed.

GASP can take account of soil non-linearity in an approximate manner by limiting the strip-soil contact pressures to not exceed the bearing capacity (in compression) or the raft uplift capacity in tension. The pile loads are similarly limited to not exceed the compressive and uplift capacities of the piles. However, the ultimate pile load capacities must be pre-determined, and are usually assumed to be the same as those for isolated piles. In reality, as shown by Katzenbach et al (1998), the loading transmitted to the soil by the raft can have a beneficial effect on the pile behaviour in the piled raft system. Thus, the assumptions involved in modelling piles in the GASP analysis will tend to be conservative.

In carrying out a nonlinear analysis in which strips in two directions are analyzed, it has been found desirable to only consider nonlinearity in one direction (the longer direction) and to consider the pile and raft behaviour in the other (shorter) direction to be linear. Such a procedure avoids unrealistic yielding of the soil beneath the strip and hence unrealistic settlement predictions.

### 5.2 Plate on Springs Approach (GARP)

In this type of analysis, the raft is represented by an elastic plate, the soil is represented by an elastic continuum and the piles are modelled as interacting springs. Some of the early approaches in this category (e.g. Hongladaromp et al, 1973) neglected some of the components of interaction and gave pile-raft stiffnesses which were too large.

Poulos (1994) has employed a finite difference method for the plate and has allowed for the various interactions via approximate elastic solutions. This analysis has been implemented via a program GARP (Geotechnical Analysis of Raft with Piles). Allowance has been made for layering of the soil profile, the effects of piles reaching their ultimate capacity (both in compression and tension), the development of bearing capacity failure below the raft, and the presence of free-field soil settlements acting on the foundation system. The approximations involved are similar to those employed in the program GASP for piled strips.

A later version of GARP (Sales et al, 2000) has replaced the finite difference analysis for the raft with a finite element analysis, and has employed a modified approach to considering the development of the ultimate load capacity in the piles.

Russo (1998) and Russo and Viggiani (1997) have described a similar approach to the above methods, in which the various interactions are obtained from elastic theory, and non-linear behaviour of the piles is considered via the assumption of a hyperbolic load-settlement curve for single piles. Pile-pile interaction is applied only to the elastic component of pile settlement, while the non-linear component of settlement of a pile is assumed to arise only from loading on that particular pile.

Most analyses of piled rafts are based on the raft being treated as a thin plate, and it is of interest to see what the effect of using thick plate theory is on the numerical predictions. Poulos et al (2001) have examined the effect of the method of modelling the raft as a thin plate who analyzed a typical problem using firstly, a three dimensional finite element program where the raft was firstly modelled using thin shell theory, and then secondly, by making the raft 0.3m thick, and assigning the raft modulus to that part of the finite element mesh representing the raft. It was assumed in the analysis that there was no slip between the raft and the soil or between the piles and the soil. It was found that there was not a great deal of difference in the computed deflections for the raft , for both a stiff raft and a flexible raft.

It was concluded that the use of thin shell elements to represent the raft will lead to reasonable estimates of deflections, and therefore moments, as long as the raft is not extremely thick. Stresses in the soil will be higher for the thin shell analysis, and this effect may become important if yield of the soil due to concentrated loads is of concern.

## 6. MORE RIGOROUS COMPUTER METHODS

### 6.1 Two – Dimensional Numerical Analysis (FLAC)

Methods in this category are exemplified by the analyses described by Desai (1974), Hewitt and Gue (1994) and Pradoso and Kulhawy (2001). In the former case, the commercially available program FLAC has been employed to model the piled raft, assuming the foundation to be a two-dimensional (plane strain) problem, or an axially symmetric three-dimensional problem. In both cases, significant approximations need to be made, especially with respect to the piles, which must be "smeared" to a wall and given an equivalent stiffness equal to the total stiffness of the piles being represented. Problems are also encountered in representing concentrated loadings in such an analysis, since these must also be smeared. Unless the problem involves uniform loading on a symmetrical raft, it may be necessary to carry out analyses for each of the directions in order to obtain estimates of the settlement profile and the raft moments. As with the plate on springs approach, this analysis cannot give torsional moments in the raft.

### 6.2 Three – Dimensional Numerical Analysis

A complete three-dimensional analysis of a piled raft foundation system can be carried out by finite element analysis (e.g. Katzenbach et al, 1998) or by use of the commercially available computer program FLAC 3D. In principle, the use of such a program removes the need for the approximate assumptions inherent in all of the above analyses. Some problems still remain, however, in relation to the modelling of the pile-soil interfaces, and whether interface element should be used. If they are, then approximations are usually involved in the assignment of joint stiffness properties. Apart from this difficulty, the main problem is the time involved in obtaining a solution, in that a non-linear analysis of a piled raft foundation can take several days, even on a modern computer running at 450 MHz. Such analyses are therefore more suited to obtaining benchmark solutions against which to compare simpler analysis methods , rather than as routine design tools.

## 7. APPLICATION TO SIMPLIFIED PROBLEM

In order to compare the predicted behaviour of a piled raft from a number of different methods, the hypothetical example in Figure 6 has been analyzed (Poulos et al, 1997). While the problem is rather simplistic, it is useful in that the inevitable differences which are involved in the assessment of parameters in real cases are avoided, and the problem involves column loading rather than merely uniformly distributed loading. The comparisons focus on the predicted behaviour of the piled raft for a given set of soil, pile and raft parameters. However, some consideration is also given to the influence on the foundation behaviour of some of the pile and raft parameters.

The methods employed, and the assumptions involved in the use of each method, are outlined below.

a) Poulos-Davis-Randolph (PDR) Method:

In applying this approach, the stiffness of the raft was computed by hand from elastic theory, assuming the raft to be an equivalent circular footing, and considering the centre of a flexible raft. The stiffness of the single piles was computed from the closed form approximate solutions of Randolph and Wroth (1978) while the group settlement ratio (used for computing the pile group stiffness) was approximated by $R_s = n^{0.5}$, where n = the number of piles.

b) Burland’s Approach

The stiffness of the raft was computed using a numerical analysis of the raft alone using the program GARP. To estimate the moments in the raft, the applied loads were reduced at each column location by 0.9 times the ultimate load capacity of the pile beneath that column (i.e. it was assumed that the full load capacity of the piles was mobilized). To estimate the settlement of the piled raft, the settlement of the raft, *under the full loads,* was obtained from the raft analysis, and then this settlement was reduced by the ratio of the stiffness of the raft to the piled raft (equation 6), as estimated from Randolph’s equations.

c) GASP Analysis (Strip on Springs)

In this analysis, the raft was divided into a series of three strips in each direction, as shown in Figure 6. Nonlinear effects were considered for the strips running in the long direction, while purely linear behaviour was assumed for the strips in the shorter direction. The stiffness of the individual piles was computed via the equations of Randolph and Wroth (1978), and simplified expressions were used to obtain the pile – pile interaction factors. For the analysis of each strip, the effects of the other strips in that direction were considered by computing the free-field settlements due to those strips, and imposing those settlements on to the strip being analyzed.

d) GARP Analysis (Plate on Springs)

The raft was modelled by a uniform plate, using a total of 273 elements and nodes. The stiffness of the piles and the pile – pile interaction factors were computed from a boundary element analysis, using the program DEFPIG (Poulos, 1990).

e) FLAC (2-D) Analysis

Using the symmetry of the problem, the soil-raft-pile system in the longer direction was analyzed, using 39 grid lines in the horizontal direction and 34 grid lines in the vertical direction. The soil was modelled as a Mohr-Coulomb material, using the undrained shear strength parameters of the soil shown in Figure 6. To obtain the pile properties, the axial stiffness of the elements representing the piles was "smeared" over a 6m width while the concentrated loads were similarly smeared. The loads were applied as uniform loadings over the elements representing the piles.

f) FLAC 3D Analysis

Because of symmetry, only the quarter-problem was modelled. The three-dimensional mesh used is shown in Figure 7 and contained 40,026 nodes and 34,468 elements. The soil was modelled as a Mohr-Coulomb material, as for the two-dimensional analysis. The pile loads were computed from the vertical stresses at the head of each pile, while the raft moments were computed from the horizontal stresses in the bottom layer of elements representing the raft.

### 7.1 Comparison of Solutions for Load-Settlement

Figure 8 compares the computed load-settlement relationships (up to a total load of 18 MN) computed from the various methods for the centre of the raft with 9 identical piles, one under each column. Burland's method is not suitable for obtaining the full load-settlement curves and was therefore excluded. There is reasonably good agreement between the computed load-settlement curves from all methods other than the FLAC2-D analysis. Even the simple Poulos-Davis-Randolph method gave results which agreed well with the FLAC3-D and GARP analyses. The FLAC3-D analysis gives a softer response than the other methods for loads in excess of about 12 MN, presumably because of the progressive development of plastic zones beneath the raft, and the consequent increasing importance of plastic deformations. However, the FLAC2-D analysis seriously over-predicted the settlements because of the implicit assumption of plane strain in the analysis. The comparisons in Figure 8 therefore suggest that plane strain analyses of piled rafts must be approached with extreme caution because the results may be misleading if the raft is essentially square or rectangular.

### 7.2 Comparison of Solutions for Piled Raft Response at a Typical Design Load

Table 2 summarizes the performance of a piled raft with 9 piles, for a typical design load of 12 MN (equivalent to an overall factor of safety of about 2 against ultimate failure). For the various methods considered, values are given for the central settlement, the settlement under a corner pile, the maximum moment in the raft, and the proportion of load carried by the raft.

The following observations are made from Table 2:

1. As indicated in Figure 8, all methods predict settlements which agree well, except FLAC 2-D, which gives almost twice the settlement of the other methods.
2. There is greater variability in the prediction of the settlement below the corner pile. As a consequence, the predicted differential settlement between the centre and corner columns varies between 3.0 mm and 11.8 mm. The latter value comes from the GASP piled strip analysis, and is likely to be excessive and inaccurate. This approach should then be used with caution for assessing differential settlements, although it appears to give a reasonable estimate for the overall settlement.
3. All methods indicate that the piles carry a substantial proportion of the load. The FLAC 2-D analysis gives generally larger values than the simpler methods, while FLAC 3-D gives a somewhat smaller proportion of pile load, possibly due to the earlier development of full load capacity than in the other methods.
4. Most of the simplified methods give maximum moments which are of a similar order. However, using the output stresses to compute the bending moments, both the FLAC 2-D and FLAC3-D analyses gave much lower moments than the simpler methods. However, as discussed below, the values derived in this way are inaccurate, as they are based on computed stresses at Gauss points within the outer elements representing the raft, and not the extreme fibre stresses in the raft.

To investigate further the effect of the method of calculating the bending moments from the FLAC3D analysis, two alternative approaches were taken:

- The computed stresses (which are computed at the Gauss points of each element) were extrapolated to the top and bottom of the raft, and averaged to remove the axial component; the moments were then computed from these stresses.
- The moments were computed from the displacements via numerical double differentiation and multiplication by the raft bending stiffness.

The results from these two approaches, and the original approach, are shown in Table 2. It can be seen that there is good agreement between the latter two approaches, although the moments are still significantly lower than the values from the other calculation methods. At least some of this difference may arise from the use of solid elements for the raft in the FLAC3D analysis, instead of the thin plate which represents the raft in the other approaches.

### 7.3 Influence of Number of Piles

One of the important uses of a piled raft analysis is to assess how many piles are required to achieve the desired performance. All of the analyses considered above are able to fulfil this function. For the present purposes, two analyses have been employed, the GARP computer analysis and the Poulos-Davis-Randolph (PDR) analysis. Figure 9 shows the computed load-settlement curves from each of these analyses, for various numbers of piles, ranging from 3 to 15. There is generally good agreement between the two analyses over the whole range of load.

Figure 10 summarizes the relationship between central settlement and number of piles (as obtained from the PDR analysis for a load of 12 MN), and the ultimate load capacity and number of piles. For the latter calculation, it is assumed that the ultimate capacity of the piles is the same as for an isolated single pile; this is likely to be a conservative assumption, especially for small numbers of piles.

It can be seen that the law of diminishing returns appears to apply here, in that the addition of a relatively few piles has a significant effect in reducing the settlement of the raft, but beyond about 15 piles, the additional reduction in settlement is very small. Clearly then, there is scope for economy in foundation design by carrying out analyses to assess the minimum number of piles to achieve the required settlement performance.

### 7.4 Effect of Varying Pile Length

For a 0.5m thick raft with 9 piles, Figure 11 shows the effect of varying the pile length on the maximum settlement, the differential settlement between the centre and outer piles, the maximum moment in the raft, and the proportion of load carried by the piles. The analyses have been carried out using the GARP program. As would be expected, the settlement, differential settlement and maximum moment all decrease with increasing pile length, while the proportion of load carried by the piles increases. By comparing Figures 10 and 11, it is clear that increasing the length of the piles is, for this case, a more effective design strategy for improving foundation performance than increasing the number of piles.

### 7.5 Effect of Raft Thickness

Figure 12 shows solutions from the program GARP for a piled raft with 9 piles supporting rafts of varying thicknesses. Except for thin rafts, the maximum settlement is not greatly affected by raft thickness, whereas the differential settlement decreases significantly with increasing raft thickness. Conversely, the maximum moment in the raft increases with increasing raft thickness. The proportion of load carried by the piles is insensitive to the raft thickness. For the case considered here, there is little or no benefit in increasing the raft thickness above about 0.8 m.

From the results presented herein, it can be concluded that increasing the raft thickness is effective primarily in reducing the differential settlement. However, it should also be noted that increasing the raft thickness may be very beneficial in resisting the punching shear from both piles and column loadings. The maximum column loading which can be supported by the raft without pile support beneath the column therefore increases with increasing raft thickness. This matter has been explored in greater detail by Poulos (2000).

## 8. THREE – DIMENSIONAL EFFECTS

Some useful insights into piled raft behaviour have been obtained by Katzenbach et al (1998) who carried out three-dimensional finite element analyses of various piled raft configurations. They used a realistic elasto-plastic soil model with dual yield surfaces and a non-associated flow rule. They analyzed a square raft containing from 1 to 49 piles, as well as a raft alone, and examined the effects of the number and relative length of the piles on the load-sharing between the piles and the raft, and the settlement reduction provided by the piles. An interaction diagram was developed, as shown in Figure 13, relating the relative settlement (ratio of the settlement of the piled raft to the raft alone) to the number of piles and their length-to-diameter ratio, L/d. This diagram clearly shows that, for a given number of piles, the relative settlement is reduced as L/d increases. It also shows that there is generally very little benefit to be obtained in using more than about 20 piles or so.

An interesting aspect of piled raft behaviour, which cannot be captured by simplified analyses such as GARP, is that the ultimate shaft friction developed by piles within a piled raft can be significantly greater than that for a single pile or a pile in a conventional pile group. This is because of the increased normal stresses generated between the soil and the pile shaft by the loading on the raft. Figure 14 shows an example of the results obtained by Katzenbach et al (1998). The piles within the piled raft foundation develop more than twice the shaft resistance of a single isolated pile or a pile within a normal pile group, with the centre piles showing the largest values. Thus, the usual design procedures for a piled raft, which assume that the ultimate pile capacity is the same as that for an isolated pile, will tend to be conservative, and the ultimate capacity of the piled raft foundation system will be greater than that assumed in design.

## 9. DESIGN FOR LOCALIZED COLUMN LOADINGS

Much of the existing literature does not consider the detailed pattern of loading applied to the foundation, but assumes uniformly distributed loading over the raft area. While this may be adequate for the preliminary stage described above, it is not adequate for considering in more detail where the piles should be located when column loadings are present. This section presents an approach which has been developed by Poulos (2001), and which allows for an assessment of the maximum column loadings which may be supported by the raft without a pile below the column.

A typical column on a raft is shown in Figure 15. There are at least four circumstances in which a pile may be needed below the column:

- If the maximum moment in the raft below the column exceeds the allowable value for the raft
- If the maximum shear in the raft below the column exceeds the allowable value for the raft
- If the maximum contact pressure below the raft exceeds the allowable design value for the soil
- If the local settlement below the column exceeds the allowable value.

To estimate the maximum moment, shear, contact pressure and local settlement caused by column loading on the raft, use can be made of the elastic solutions summarized by Selvadurai (1979). These are for the ideal case of a single concentrated load on a semi-infinite elastic raft supported by a homogeneous elastic layer of great depth, but they do at least provide a rational basis for design. It is possible also to transform approximately a more realistic layered soil profile into an equivalent homogeneous soil layer by using the approach described by Fraser and Wardle (1976). Figure 15 shows the definition of the problem addressed, and a typical column for which the piling requirements (if any) are being assessed..

(a) Maximum moment criterion:

The maximum moments $M_x$ and $M_y$ below a column of radius c acting on a semi-infinite raft are given by the following approximations:

$$M_x = A_x \cdot P \qquad (7a)$$

$$M_y = B_y \cdot P \qquad (7b)$$

where $A_x$ = $[A - 0.0928 ( \ln (c / a))]$
$B_y$ = $[B - 0.0928 ( \ln (c / a))]$
A, B = coefficients depending on $\delta/a$
$\xi$ = distance of the column centre line from the raft edge
a = characteristic length of raft
$= t \cdot [E_r \cdot (1-\nu_s^2) / 6. E_s. (1-\nu_r^2)]^{1/3}$ (7c)
t = raft thickness
$E_r$ = raft Youngs modulus
$E_s$ = soil Youngs modulus
$\nu_r$ = raft Poissons ratio
$\nu_s$ = soil Poissons ratio.

The coefficients A and B are plotted in Figure 16 as a function of the distance x.

The maximum column load, $P_{c1}$, that can be carried by the raft without exceeding the allowable moment is then given by:

$$P_{c1} = M_d / (\text{larger of } A_x \text{ and } B_y) \tag{8}$$

where $M_d$ = design moment capacity of raft.

(b) Maximum Shear Criterion

The maximum shear $V_{max}$ below a column can be expressed as:

$$V_{max} = (P - q\,\pi\,c^2).\,C_q / 2\pi c \tag{9}$$

where q = contact pressure below raft
c = column radius
$C_q$ = shear factor, plotted in Figure 17.

Thus, if the design shear capacity of the raft is $V_d$, the maximum column load, $P_{c2}$, which can be applied to the raft is:

$$P_{c2} = V_d.\,2\pi c / C_q + q_d\,\pi\,c^2 \tag{10}$$

where $q_d$ = design allowable bearing pressure below raft.

(c) Maximum Contact Pressure Criterion

The maximum contact pressure on the base of the raft, $q_{max}$, can be estimated as follows:

$$q_{max} = \bar{q}\,.\,P / a^2 \tag{11}$$

where $\bar{q}$ = factor plotted in Figure 18
a = characteristic length defined in Equation 7c .

The maximum column load, $P_{c3}$, which can be applied without exceeding the allowable contact pressure is then :

$$P_{c3} = q_u\,a^2 / (F_s.\ \bar{q}) \tag{12}$$

where $q_u$ = ultimate bearing capacity of soil below raft
$F_s$ = factor of safety for contact pressure..

(d) Local Settlement Criterion

The settlement below a column (considered as a concentrated load) is given by:

$$S = \omega\,(1-\nu_s^2)\,P / (E_s\,.\,a) \tag{13}$$

where ω = settlement factor plotted in Figure 19.

It should be recognized that this expression does not allow for the effects of adjacent columns on the settlement of the column being considered, and so is a local settlement which is superimposed on a more general settlement “bowl”.

If the allowable local settlement is $S_a$, then the maximum column load, $P_{c4}$, so as not to exceed this value is then:

$$P_{c4} = S_a E_s a / (\omega (1-\nu_s^2)) \quad (14)$$

(e) Assessment of Pile Requirements for a Column Location

If the actual design column load at a particular location is $P_c$, then a pile will be required if Pc exceeds the least value of the above four criteria, that is, if:

$$P_c > P_{crit} \quad (15)$$

where $P_{crit}$ = minimum of $P_{c1}$, $P_{c2}$, $P_{c3}$, or $P_{c4}$.

If the critical criterion is maximum moment, shear or contact pressure (i.e. $P_{crit}$ is $P_{c1}$, $P_{c2}$ or $P_{c3}$), then the pile should be designed to provide the deficiency in load capacity. Burland (1995) has suggested that only about 90% of the ultimate pile load capacity should be considered as being mobilized below a piled raft system. On this basis, the ultimate pile load capacity, $P_{ud}$, at the column location is then given by:

$$P_{ud} = 1.11\ F_p.\ [\ P_c - P_{crit}] \quad (16)$$

where $F_p$ = factor of safety for piles.

When designing the piles as settlement reducers, $F_p$ can be taken as unity.

If the critical criterion is local settlement, then the pile should be designed to provide an appropriate additional stiffness. For a maximum local settlement of $S_a$, the target stiffness, $K_{cd}$, of the foundation below the column is:

$$K_{cd} = P_c / S_a \quad (17)$$

As a first approximation, using Equation 1, the required pile stiffness $K_p$ to achieve this target stiffness can be obtained by solving the following quadratic equation:

$$K_p^2 + K_p\ [\ K_r\ (1-2\alpha_{cp}) - K_{cd}] + \alpha_{cp}^2.\ K_r.\ K_{cd} = 0 \quad (18)$$

where $\alpha_{cp}$ = raft-pile interaction factor
$K_r$ = stiffness of raft around the column.

$\alpha_{cp}$ can be computed from Equation 3, while the raft stiffness $K_r$ can be estimated as the stiffness of a circular foundation having a radius equal to the characteristic length a (provided that this does not lead to a total raft area which exceeds the actual area of the raft.

## 9.1 Example Case

To illustrate the maximum column loads which are computed by the approach outlined above, an example has been considered in which a raft of thickness t is located on a deep clay layer having a Young's modulus $E_s$. Typical design strengths and steel reinforcement are adopted for the concrete of the raft (see Figure 20), and design values of maximum moment and shear have been computed accordingly. The design criterion for maximum contact pressure has been take to be a factor of safety $F_s$ of 1.2, while the local settlement is to be limited to 20 mm. An interior column, well away from the edge of the raft, is assumed.

Figure 20 shows the computed maximum loads for the four criteria, as a function of raft thickness and soil Young's modulus. The following observations are made:

- For all design criteria, the maximum column load which may be sustained by the raft alone increases markedly with increasing raft thickness
- The maximum columns loads for bending moment and shear requirements are not very sensitive to the soil Young's modulus, whereas the maximum columns loads for the contact pressure and local settlement criteria are highly dependent on soil modulus
- For the case considered, the criteria most likely to be critical are the maximum moment and the local settlement.

Although the results in Figure 20 are for a hypothetical case, they nevertheless give a useful indication of the order of magnitude of the maximum column loads which the raft can sustain and the requirements for piles which may need to provided at a column location. For example, if a 0.5 m thick raft is located on a soil with Young's modulus of 25 MPa, the lowest value of column load is found to be about 2.8 MN (this occurs for the maximum moment criterion). If the actual column load is 4 MN, then from Equation 14, if $F_p$ is taken as unity, the required ultimate load capacity of the pile would be 1.11 (4.0-2.8) = 1.33 MN.

## 10. PILED RAFTS SUBJECTED TO GENERAL LOADINGS

### 10.1 Introduction

All of the methods mentioned previously deal only with piled raft foundations subjected to vertical loading or moments, but not horizontal loads. Based on finite layer theory, Ta & Small (1996) developed a method of analysing a piled raft (on or off the ground) subjected to vertical loads, and on the basis of previous results, an approximate method was then introduced in order to save computer running time (Ta & Small 1997). Zhang & Small (2000) subsequently developed a method of analysing piled rafts subjected to both horizontal and vertical loads where the raft is clear of the ground.

Following this work, Small and Zhang (2000) have developed a new method for the analysis of piled rafts has been developed based on finite layer theory, a method developed for the analysis of horizontally layered materials (Small & Booker 1986). The raft is supported by both the soil and the piles, and it can be subjected to horizontal and vertical loads as well as moments. The movements of the piled raft in three directions (x, y, z) and rotations in two directions (x, y) may be computed by the program APRAF developed by Zhang and Small (2000). Use of this program has shown that:

1. the program requires only a small amount of computer memory,
2. it can deal with loads and moments applied to a piled raft in all directions,
3. all the loads applied to the piled raft and displacements are coupled,
4. computing time is relatively small compared to alternative numerical techniques (i.e. finite element or finite difference methods).

### 10.2 Method Of Analysis

As shown in Figure 21, the piled raft may be separated into an isolated raft which is subjected to external loading {Q} and interface forces {$P_r$}, and a pile group, embedded in a layered soil, subjected to interfaces forces {$P_{sp}$}. The forces between the piles and layered soil can be treated as a series of ring loads applied to 'nodes' along the pile shaft (Zhang & Small 1999). These loads are both horizontal and vertical, and if enough are used, they can approximate the continuous forces that act along the pile shaft reasonably well.

The raft is divided into a series of rectangular elements with each pile head assumed to fit within one of the raft elements. The raft is modelled as a thin plate and each element has four nodes and twenty-four degrees of freedom. The interface force applied to any of the raft elements is assumed to be a uniform load over the element.

The piles and the soil are subjected to interface forces transferred from the raft and may be analysed with the method developed by Zhang & Small (1999). The forces acting on the pile heads are assumed to be concentrated loadings and the forces applied to the soil surface are taken to be a series of rectangular blocks of uniform pressure. The displacements of the layered soil and pile heads can then be computed. Torsional loadings are not considered on the pile heads, and so the analysis is limited to where torsion is not of major concern.

*Analysis of raft*

In the analysis of the raft, some nodes on the raft must be restrained from undergoing free body rotations and translations. In the present paper two corner nodes of the raft were chosen as points of restraint where one is completely fixed in all directions (i.e. six freedoms) and the other is fixed only in the y direction to resist rotation of the raft about the z-axis. The rigid body translations and rotations about the first pinned node of the raft are assumed to be $D_x$, $D_y$, $D_z$, $\theta_x$, $\theta_y$ and $\theta_z$. Therefore, the actual displacement $\{\delta_r\}$ at the centre of each raft element may be expressed as

$$\{\delta_r\} = [I_r]\{P_r\} + \{a\}D_x + \{b\}D_y + \{c\}D_z + \{d\}\theta_x + \{e\}\theta_y + \{f\}\theta_z + \{\delta_{r0}\} \quad (19)$$

where $[I_r]$ = influence matrix of the pinned raft; $\{P_r\}$ = the vector of interface loads and moments on the raft elements; $\{\delta_{r0}\}$ = displacements at the centres of the raft elements due to applied loads on the pinned raft; and $\{a\}$ to $\{f\}$ are auxiliary vectors related to the raft geometry.

*Analysis of pile group*

In the analysis of a pile group embedded in a layered soil, the following interactions must be taken into account: soil-to-soil, soil-to-pile, pile-to-pile and pile-to-soil. The interaction between soil and soil may be directly solved by the finite layer method developed by Small & Booker (1986) and the other interactions may be obtained by using the method developed by Zhang & Small (1999) combined with the finite layer method. The displacements at the top of each pile and the centre of each soil surface element under the interface forces transferred from the raft can be expressed as

$$\{\delta_{sp}\} = \lfloor I_{sp} \rfloor \{P_{sp}\} \quad (20)$$

where $\lfloor I_{sp} \rfloor$ = influence matrix of the pile enhanced soil continuum; $\{P_{sp}\}$ = interface load vector between the raft and the pile-enhanced soil; and $\{\delta_{sp}\}$ = vector of interface displacement between the raft and the pile-enhanced soil.

*Analysis of piled raft*

By considering the compatibility of displacements and the equilibrium of interaction forces between the raft and the soil surface and pile heads, we may obtain

$$\{\delta_r\} = \{\delta_{sp}\} \quad (21)$$

$$\{P_r\} = -\{P_{sp}\} \quad (22)$$

Combination of equations (19) to (23) leads to

$$\left(\left[I_r\right]+\left[I_{sp}\right]\right)\left\{P_{sp}\right\}-\{a\}D_x-\{b\}D_y-\{c\}D_z-\{d\}\theta_x-\{e\}\theta_y-\{f\}\theta_z=\left\{\delta_{r0}\right\} \quad (23)$$

Taking into account the equilibrium of applied forces and interface forces acting on the raft gives

$$\left\{a^{'}\right\}\left\{P_{sp}\right\}=P_x \quad (23)$$

$$\left\{b^{'}\right\}\left\{P_{sp}\right\}=P_y \quad (25)$$

$$\left\{c^{'}\right\}\left\{P_{sp}\right\}=P_z \quad (26)$$

$$\left\{d^{'}\right\}\left\{P_{sp}\right\}=M_x \quad (27)$$

$$\left\{e^{'}\right\}\left\{P_{sp}\right\}=M_y \quad (28)$$

$$\left\{f^{'}\right\}\left\{P_{sp}\right\}=M_z \quad (29)$$

where $P_x$, $P_y$, $P_z$ are the total loads applied to the raft in the x, y and z directions; $M_x$, $M_y$ are the total moments applied to the raft about the pinned point; $M_z$ is the total moment about the z-axis (at the first pin) due to $P_x$ and $P_y$; and $\left\{a^{'}\right\}$ to $\left\{f^{'}\right\}$ are auxiliary vectors related to vectors $\{a\}$ to $\{f\}$.

Solving equations (23) to (29) will give the interface pressures on the pile-enhanced soil and solutions for the displacements in the raft may be obtained by substituting the pressures into equation (19).

### 10.3 Comparisons with results of large scale test of a 16-pile group

Small and Zhang (2000) have compared their computed results with the results of field tests carried out by Ruesta & Townsend (1997). An isolated single pile and a large-scale group with a spacing ratio of 3 were tested under horizontal loading. The pile cross-section as shown in Figure 22 consisted of a 350 mm diameter steel pipe (9.5 mm thick) embedded in concrete such that the pile diameter was 0.76 m. The ratio of the embedded pile length to the pile diameter is 18.42. The moduli of the concrete and the pipe were given as 34,475 MPa and 190,302 MPa, respectively. From the following equation

$$E_{concrete}I_{concrete} + E_{pipe}I_{pipe} = E_{equiv}I_{equiv}$$

the equivalent modulus of the piles may be obtained as 43,660 MPa. The soil consists of two layers, an upper layer of sand 4 m thick and a lower layer of cemented sand 10 m thick. The Poisson's ratio of the soil is taken as 0.35. According to field dilatometer and pressuremeter tests, the modulus of the upper layer increases with depth and as it has less effect on the pile behaviour, the lower layer is assumed to have a homogeneous modulus (as shown in Fig. 22). The soil modulus was determined by the back-analysis of the single pile. Firstly, according to the results of the in-situ dilatometer and pressuremeter tests, the variation of the soil modulus with depth may be approximately determined as shown in Figure 4. In back-analysis the soil moduli at z = 0.0 m and z = 4.0 m were chosen by linearly changing the measured modulus. The final result backfigured after obtaining an acceptable fit to single pile test data shows that the modulus of the upper layer varies from 12.35 MPa at the ground surface to 23.35 MPa at a depth of 4 meters, and below that depth, the soil modulus is 190.5 MPa on average. By using the back-analyzed modulus, the displacement of each pile in the pile group as shown in Figure 23 may be calculated from the present program APRAF, where the raft is assumed to be flexible (i.e. an extremely low modulus is used to model the pile cap as the piles were not connected at the heads).

The moment in the pile predicted by the present method was also compared with the measured moment. The measured average moment in the piles located in each row has been plotted in Figures 24 and 25 separately. It should be noted that the average moment in the leading row of piles is exactly the same as that of the trailing row in theory when the same loading is applied to each pile in the group. Therefore, there is only one bending moment curve for both leading and trailing rows, and one curve for the middle leading row and middle trailing row as shown in Figures 24 and 25. The figures demonstrate that the predicted moments are in good agreement with the measured moments for each row of piles. It was observed that the difference between the predicted maximum moment and measured maximum moment is about 12.9% for the leading and trailing rows and only about 7.5% for the middle trailing row and middle leading row.

### 10.4 Parametric Study

Small and Zhang (2000) have carried out a parametric study of a square piled raft foundation with 16 (4×4) piles embedded in a deep uniform soil. Poisson's ratios of the raft and soil were chosen to be 0.15 and 0.35, respectively. Both the thickness of the raft and the diameters of the piles were taken as 0.5 m. The horizontal and vertical displacements at the top of pile 1 were used in the plots of Figures 27, 29 and 31.

For both the horizontal and vertical loading cases, normalised lateral and vertical displacements $I_{uxx}$ and $I_{uzz}$ can be expressed as

$$I_{uxx} = \frac{E_s D}{q_x B_r L_r} u_x \qquad (30)$$

$$I_{uzz} = \frac{E_s D}{q_z B_r L_r} u_z \qquad (31)$$

where $u_x$ and $u_z$ are the actual horizontal and vertical displacements, respectively; $E_s$ is the soil modulus; D is the pile diameter; $q_x$ and $q_z$ are the uniform lateral and vertical loads; and $B_r$ and $L_r$ are the breadth and length of the raft in plan. The overhang of the raft (around the perimeter) was one pile diameter.

*Effect of pile-soil stiffness ratio on displacement and load distribution*

In the first example, the pile spacing ratio S/D was chosen to be 5, the soil modulus as 10 MPa and the ratio of the raft modulus to the soil modulus, $E_r/E_s$, was taken as 2000. The pile slenderness ratio L/D was chosen to be 30 and the soil depth was assumed to be infinite. The results of the analysis are shown in Figures 26 to 28 for different pile-soil stiffness ratios, for the case of a uniform shear loading $\tau$ applied to the raft in the x-direction.

Figure 26 shows the variation of shear pressures on the interface along Section A-A. It may be seen that the highest values of shear stress occur at the positions of the piles. Moreover, an increase in the pile-soil stiffness ratio leads to an obvious increase in the shear pressures on the pile heads and slightly decreases the shear pressures on the soil surface.

Figure 27 shows that, as expected, an increase of the pile-soil stiffness ratio ($E_p/E_s$) leads to a reduction in the horizontal displacement of the piled raft under horizontal load.

For the piled raft under vertical loading, reduction in vertical displacement also occurs. However, when the pile-soil stiffness ratio is lower, the vertical displacement reduces fairly rapidly with pile-soil stiffness ratio, but when the pile-soil stiffness ratio is in excess of 1000, the vertical deflection of the piled raft is not very sensitive to the pile-soil stiffness ratio.

It is also of interest to see if the piles carry most of the load, or whether the raft carries the load for the examples considered here. The results of the analyses are presented in Figure 10 which shows that the percentage of the horizontal load carried by the piles increases as the pile-soil stiffness ratio increases (for the range of ratios considered) while the vertical load carried by the piles stops increasing when the pile-soil stiffness ratio exceeds 1000.

*Effect of raft-soil stiffness ratio on displacement and load distribution*

To examine the effect of the raft-soil stiffness ratio $E_r/E_s$, the pile spacing ratio S/D, the pile slenderness ratio L/D and the soil modulus $E_s$ were kept constant while the ratio of the pile modulus to the soil modulus $E_p/E_s$ was chosen to be 2000. As the stiffness of the raft also depends on the raft thickness it is necessary to know the raft thickness $t_r$ as shown on Figures 29 and 30. The normalised displacement of the piled raft (corner pile) and the load carried by the piles are plotted in Figures 11 and 12 versus the raft-soil stiffness ratios.

Figure 29 shows the raft-soil stiffness ratio has only a limited influence on the displacement of the piled raft whether it is subjected to horizontal loading or vertical loading. This is unlike the effect of pile-soil stiffness that has a large effect on displacements.

However, from Figure 30 it may be seen that for a lower raft-soil stiffness ratio (less than 100 in this example) increase in the raft-soil stiffness ratio will lead to an obvious rise in the percentage of loading carried by piles. For higher raft-soil stiffness ratios (i.e. greater than 100) the variation of the raft-soil stiffness ratio will have only a small effect on the loading distribution.

### *Effect of pile spacing ratio on displacement and load distribution*

The effect of pile spacing was next examined. As the pile spacing becomes larger so does the pile cap or raft, and so the total applied load also increases (as the load is uniformly distributed). Keeping the pile slenderness ratio L/D = 30 and the soil modulus $E_s$ = 10 MPa, and both $E_p/E_s$ and $E_r/E_s$ equal to 2000; the computed results are shown in Figures 31 and 32.

As seen from Figure 31, the pile spacing ratio has a pronounced effect on the displacement of the piled raft whether the piled raft is subjected to horizontal loading or vertical loading, especially for small pile spacing ratios. Increase in pile spacing ratio can also lead to significant reduction of the horizontal loading carried by the piles as shown in Figure 32. However, under such conditions ($E_p/E_s = E_r/E_s = 2000$), the pile spacing ratio has only a small influence on the percentage of the vertical loading carried by the piles (Fig. 32).

### 10.5 Summary

A method for analysing the behaviour of piled rafts constructed in elastic soils has been developed by Small and Zhang (2000), and this shows good agreement with solutions derived from previously-existing programs and also full-scale pile group test results. The method may be used for the analysis of piled rafts with general type loadings and can consider a raft in contact with the ground. It can also be used for problems where the soil modulus varies from layer to layer. Furthermore, the method has the advantage that the data is easy to prepare and does not involve creating large meshes as would be required for finite element solutions.

## 11. THE IMPORTANCE OF CONSIDERING PILE-RAFT INTERACTION

Many analysts of piled raft foundations employ structural analysis programs in which the raft is represented by a plate and the piles as springs. It is common for the spring stiffness of the piles to be computed for a single isolated pile, ignoring the effects of pile-soil-pile interaction. It is also common for such analyses to ignore the effects of raft-pile and pile-raft interaction. Such analyses will therefore tend to give a foundation stiffness which is too large, and settlements which are too low.

The importance of considering interaction effects can be most easily gauged via the simplified PDR analysis. If pile-soil-pile interaction is ignored, the stiffness of a group of n piles is n times the stiffness of a single pile. If raft-pile and pile-raft interaction is ignored, the interaction factor $\alpha_{cp}$ in Equation 3 will be zero.

For the simple problem shown in Figure 6, the stiffness of the piled raft has been computed by the simple PDR method for three cases:

1. With proper consideration of the pile-soil-pile and raft-pile interactions outlined above.
2. With consideration of the interaction within the pile group, but ignoring raft-pile interaction.
3. Without consideration of any interactions, i.e. adding the stiffnesses of the raft and each of the individual piles.

The results of the analysis are summarized in Table 3, which reveals that there is an extremely large unconservative error in simply adding the stiffnesses of the raft and each of the piles in the pile group (Case 3). For a raft with 15 piles, the foundation stiffness is over-estimated by more than 200%. If account is taken of the interaction among the piles in the group, but the raft-pile interaction is ignored, the errors are still significant and unconservative, although much less serious than if all the stiffnesses are simply added.

Thus, it appears that the use of structural programs for piled raft analyses, without due consideration of the interactions involved in the piled raft system, may lead to serious underestimates of settlement. As a by-product, they may also lead to inaccurate estimates of raft bending moments and pile loads.

## 12. APPLICATION TO PRACTICAL DESIGN

In applying piled raft analyses to practical foundation design, it is suggested that a 3-stage design process can be adopted:

1. The simple PDR method can be used initially to assess approximately the required number of piles to satisfy the overall bearing capacity and settlement requirements.
2. A simple approach (for example, as described in Section 9) can be used to assess the maximum column load which the raft can sustain without a pile. This will provide a means of assessing under which columns piles are required, for a particular raft thickness, and the requirements for such piles.
3. A detailed analysis for final design, using numerical analyses such as GARP or FLAC 3D, to provide detailed estimates of settlement and differential settlement under various loading combinations, and also details of the raft and pile behaviour for structural design.

The location of the piles will depend on the pattern of loading and the presence or otherwise of concentrated column loadings. Normally, piles will be necessary under relatively heavy column loadings when the raft thickness is not sufficient to provide the necessary shear and moment resistance, or when the localized settlement is excessive. In addition, the presence of high lateral loadings, for example, due to wind, may require that piles be placed near the edges of the raft, even though under normal serviceability loadings, the settlements near the edges may not be large. The effects of the lateral loadings themselves may also need to be analyzed, using an analysis of the type outlined in Section 10.

If the loading is relatively uniformly distributed and the lateral loadings are not large (e.g. for a storage tank), considerable economy can be achieved by concentrating the piles near the centre of the raft. Horikoshi and Randolph (1998), de Sanctis et al (2001) and Viggiani (2001) give useful guidelines for such cases.

For design applications, it is essential to obtain a reasonable assessment of the geotechnical parameters for the subsoil profile. In many cases, this may involve the use of appropriate correlations between SPT or CPT values and soil modulus and strength. Decourt (1995) provides some useful correlations with SPT data, and these are summarized also by Poulos (2000), who also describes a number of practical applications of piled raft foundations.

## 13. CONCLUSIONS

A variety of methods exist for the analysis of piled raft foundation systems, ranging from relatively simple methods which can be implemented with minimal computer requirements, through to complex three-dimensional finite element or FLAC3D analyses. A comparison of some of these methods made for a very simple idealized problem has revealed that most give similar results for the maximum settlement and the load sharing between the piles and the raft. There is however a greater spread of results with respect to differential settlements and bending moments in the raft.

Some of the conclusions which emerge from the work summarized in this report may be summarized as follows:

1. Simple methods can be used with some confidence for preliminary design purposes, with the more complex analyses being left for the detailed design stage.
2. Two-dimensional analyses, such as FLAC2-D, may lead to serious over-estimates of settlement and pile loads because of the plane strain assumptions which are inherently present.
3. Three-dimensional analyses, such as FLAC 3-D, are potentially the most accurate numerical methods available for piled raft analysis. They are however very time-consuming to set up and run, and also may lead to unsatisfactory (and unconservative) bending moments if solid elements are used for the raft and the output stresses are used directly to compute the moments. More satisfactory results are obtained by extrapolating the stresses at the Gauss points or by using the computed displacements to obtain the moments.
4. It is essential to take account of the various interactions which exist within a piled raft foundation: pile-pile, pile-raft, raft-pile, and raft-raft. These interactions are usually ignored in most conventional structural analyses, which may then seriously underestimate the settlement and differential settlement, and also the amount of load carried by the raft.
5. A method has now been developed for analyzing piled rafts subjected to lateral, as well as vertical, loading. This method, while involving some simplifications, appears to be capable of predicting reasonably well the behaviour observed in a full-scale field test. Nevertheless, considerable further research is warranted to develop simplified approaches which can be used in routine design, without the need for complex numerical analyses.

Piled raft foundations have the potential to provide economical foundation systems, under the appropriate geotechnical conditions. The design philosophy should be based on both ultimate load capacity and settlement criteria, with the key question to be answered being: “what is the minimum number of piles required to be added to the raft such that the ultimate load, settlement and differential settlement criteria are satisfied?” Use of some of the methods outlined in this report can be used to assist the foundation designer to provide a rational answer to this question.

## ACKNOWLEDGEMENTS

The consideration of piled raft foundations as a key subject for study by ISSMGE Technical Committee TC18 has been stimulated and driven by Prof. Dr. W. van Impe, the Chairman of TC18 from 1994. Professor Mark Randolph has made many pioneering contributions and assisted greatly in the understanding of this type of foundation. Among the researchers who have contributed to the research effort at the University of Sydney are Associate Professor John Small, who has developed many valuable tools for analysis in recent years, Dr. L.D. Ta, Dr. J. Sinha, Dr. H.H. Zhang, Mr. Gilbert Wong, and Dr. M. M. Sales and Prof. Renato Cunha, visiting researchers from the University of Brasilia.

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## TABLE 1

Summary of Capabilities of Various Methods

| **Method** | **Response Characteristics** | | | | | **Problem Modelling** | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **Settlement** | **Diff. Settl.** | **Pile Loads** | **Raft BM** | **Tors. Shear** | **Nonlin. Soil** | **Nonlin. Pile** | **Nonunif Soil** | **Raft Flexib.** |
| Poulos & Davis (1980) | x | | | | | | x | | |
| Randolph (1983) | x | | x | | | | | | |
| Van Impe & Clerq (1995) | x | x | | | | | | | |
| Equivalent Raft (Poulos, 1994) | x | x | | | | | | | |
| Brown & Wiesner (1975) | x | x | x | x | | | | | x |
| Clancy & Randolph (1993) | x | x | x | x | x | | | x | x |
| Poulos (1994) | x | x | x | x | x | x | x | x | x |
| Kuwabara (1989) | x | | x | | | | | | |
| Hain & Lee (1978) | x | x | x | x | x | | x | | x |
| Sinha (1996) | x | x | x | x | x | x | x | | x |
| Franke et al (1994) | x | x | x | x | x | | x | | x |
| Hooper (1973) | x | x | x | x | | x | | x | x |
| Hewitt & Gue (1994) | x | x | x | x | | | | x | x |
| Lee et al (1993) | x | x | x | x | x | | | x | x |
| Ta & Small (1996) | x | x | x | x | x | | | x | x |
| Wang (1995) | x | x | x | x | x | x | x | x | x |
| Katzenbach et al (1998) | x | x | x | x | x | x | x | x | x |

### Table 2

### Summary of Computed Piled Raft Behaviour for Total Load = 12 MN

| Method | Central Settlement mm | Corner Pile Settlement mm | Maximum Raft Moment MNm/m | Percentage of Load Taken by Piles |
|---|---|---|---|---|
| Poulos-Davis-Randolph | 36.8 | - | - | 77.0 |
| GARP5 | 34.2 | 26.0 | 0.684 | 65.1 |
| GASP | 33.8 | 22.0 | 0.563 | 65.5 |
| Burland | 33.8 | 29.7 | 0.688 | 65.5 |
| FLAC 2-D | 65.9 | 60.5 | 0.284 | 79.5 |
| FLAC 3-D | 39.9 | 35.8 | (see below) | 58.2 |
| -moments directly from output stresses | | | 0.326 | |
| -moments from extrapolated stresses | | | 0.421 | |
| -moments from displacements | | | 0.484 | |

### Table 3

### Effects of Ignoring Pile-Raft Interaction

| Number of Piles | Foundation Stiffness Including Pile Group and Raft-Pile Interaction MN/m | Foundation Stiffness, Including Pile Group Interaction, but not Raft-Pile Interaction MN/m | Foundation Stiffness, Ignoring all Interactions MN/m | % Error in Considering Pile Group Interaction, but not Raft-Pile Interaction | % Error in Ignoring All Interactions |
|---|---|---|---|---|---|
| 0 | 176.0 | 176.0 | 176.0 | 0 | 0 |
| 1 | 211.2 | 292.0 | 292.0 | +38 | +38 |
| 3 | 264.6 | 376.9 | 524.0 | +42 | +98 |
| 9 | 384.6 | 524.0 | 1220.0 | +36 | +133 |
| 15 | 475.7 | 625.3 | 1916. | +31 | +206 |

# FIGURES

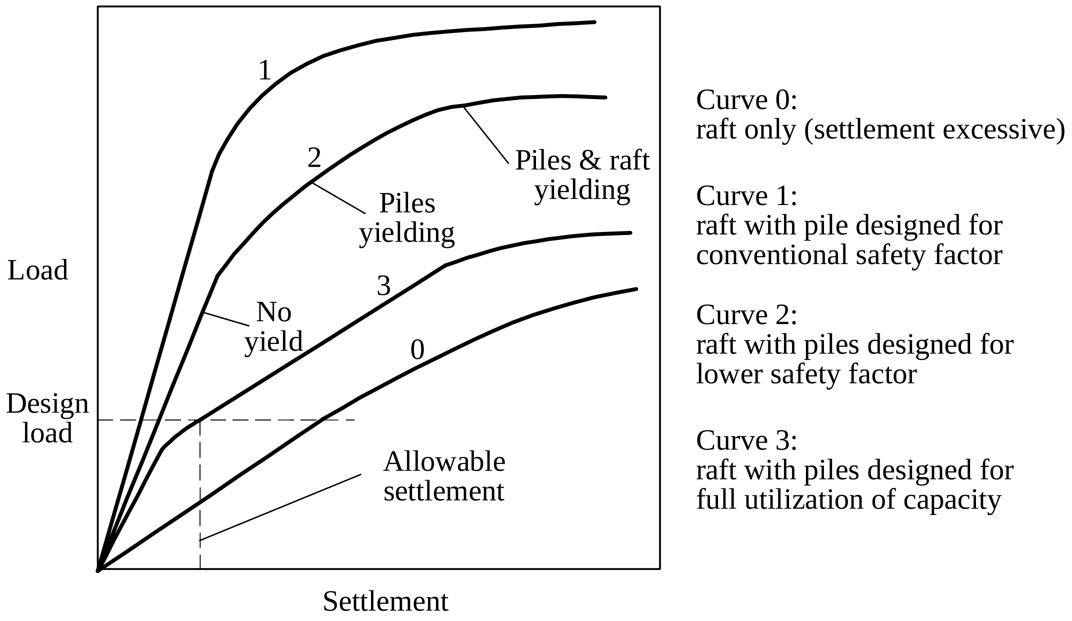


Figure 1. Load settlement curves for piled rafts according to various design philosophies.

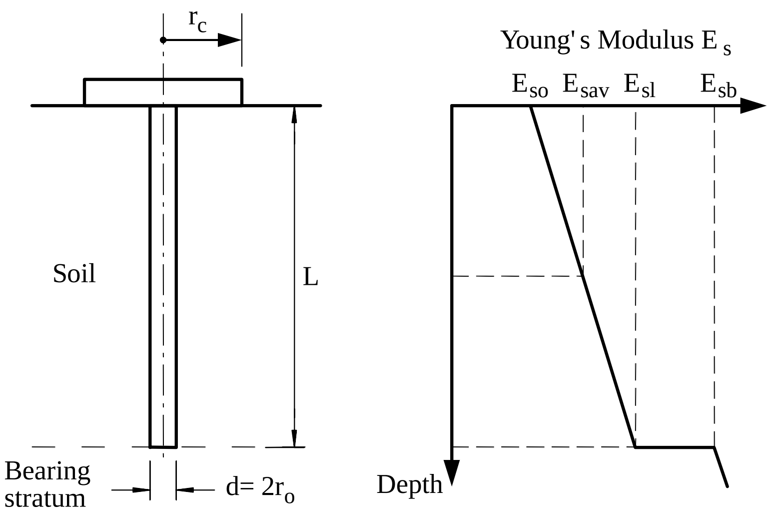


Figure 2. Simplified representation of a pile-raft unit.

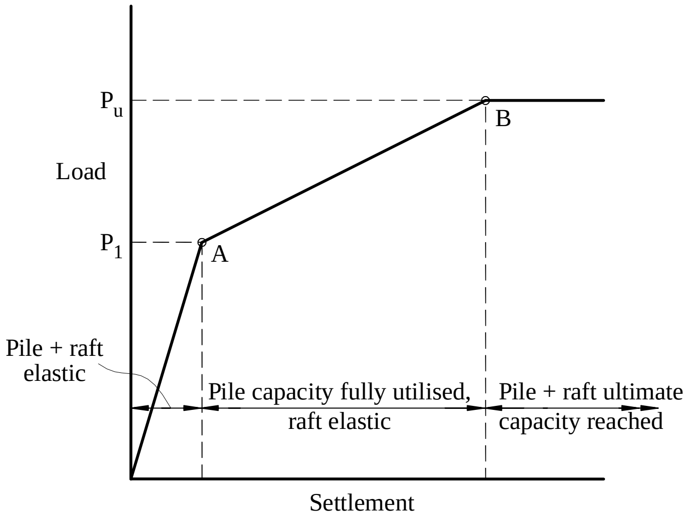


Figure 3. Simplified load-settlement curve for preliminary analysis.

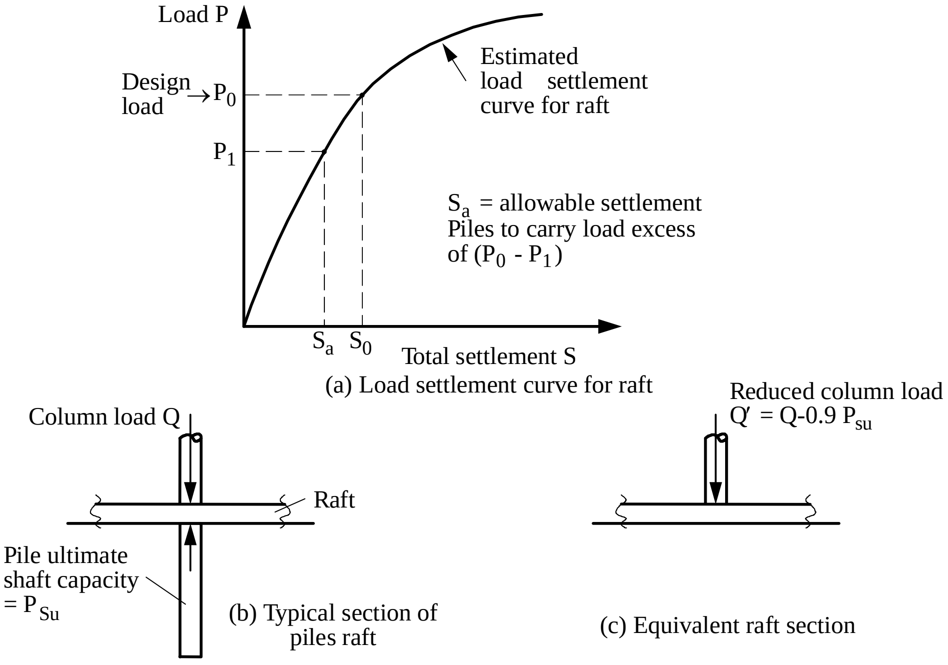


Figure 4. Burland’s simplified design concept.

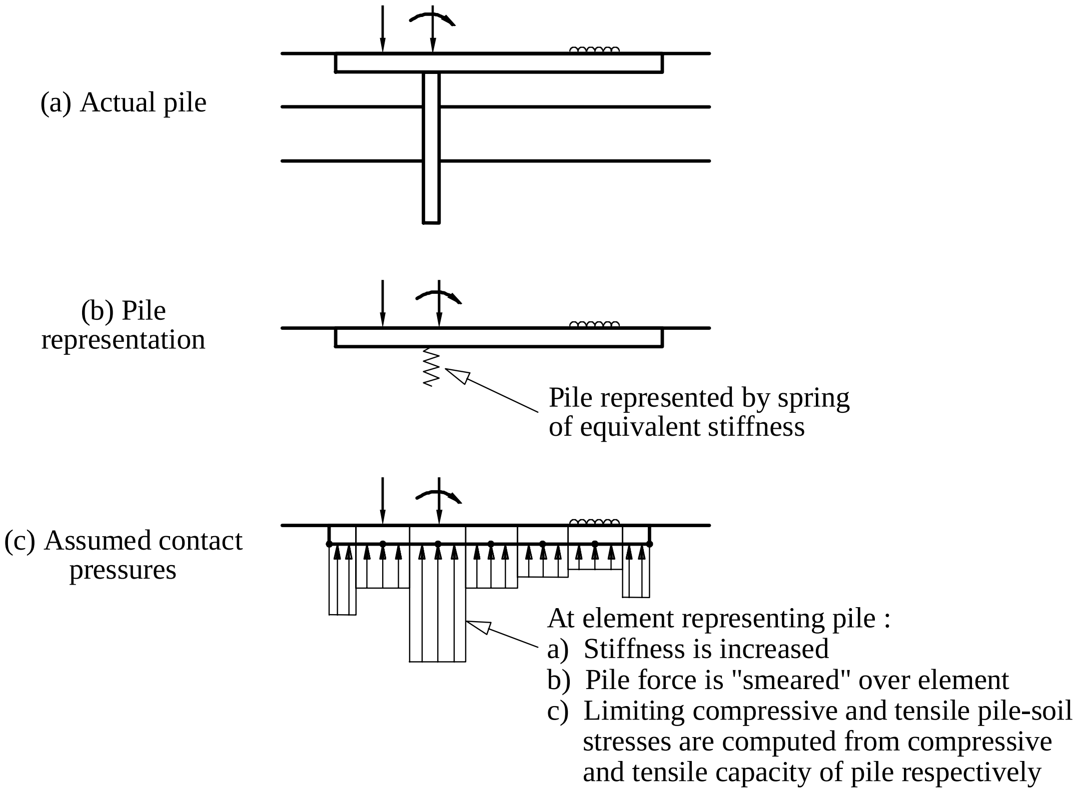


Figure 5. Representation of piled strip problem via GASP analysis (Poulos, 1991).

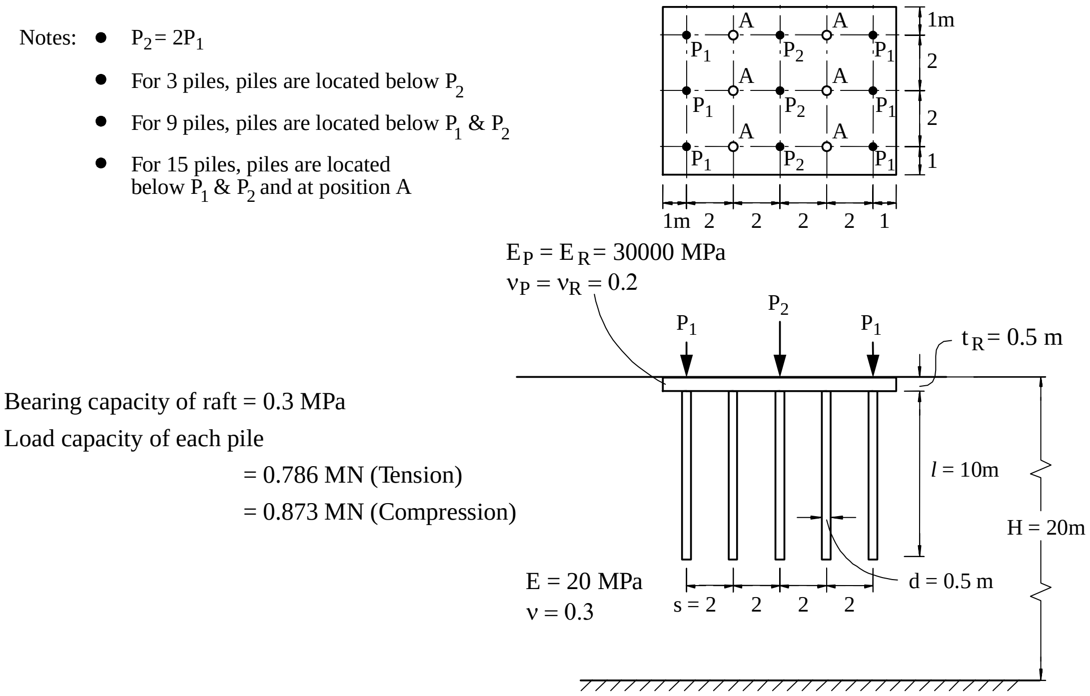


Figure 6. Simple example analysed by various methods.

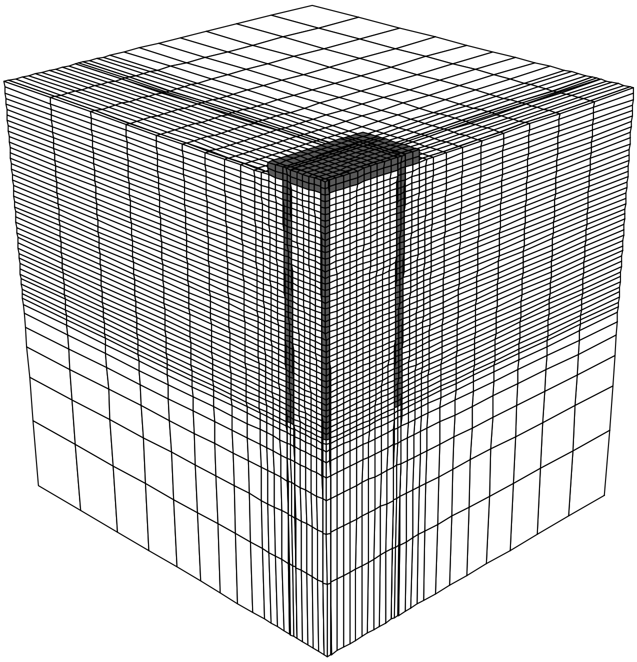


Figure 7. FLAC3D model for analysis of piled raft example.

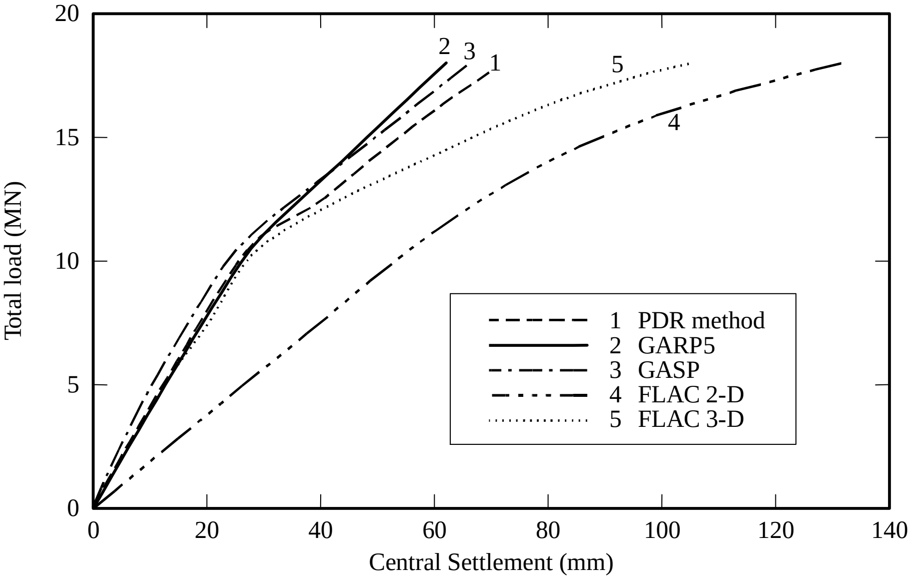


Figure 8. Comparison of various methods for load-settlement analysis.

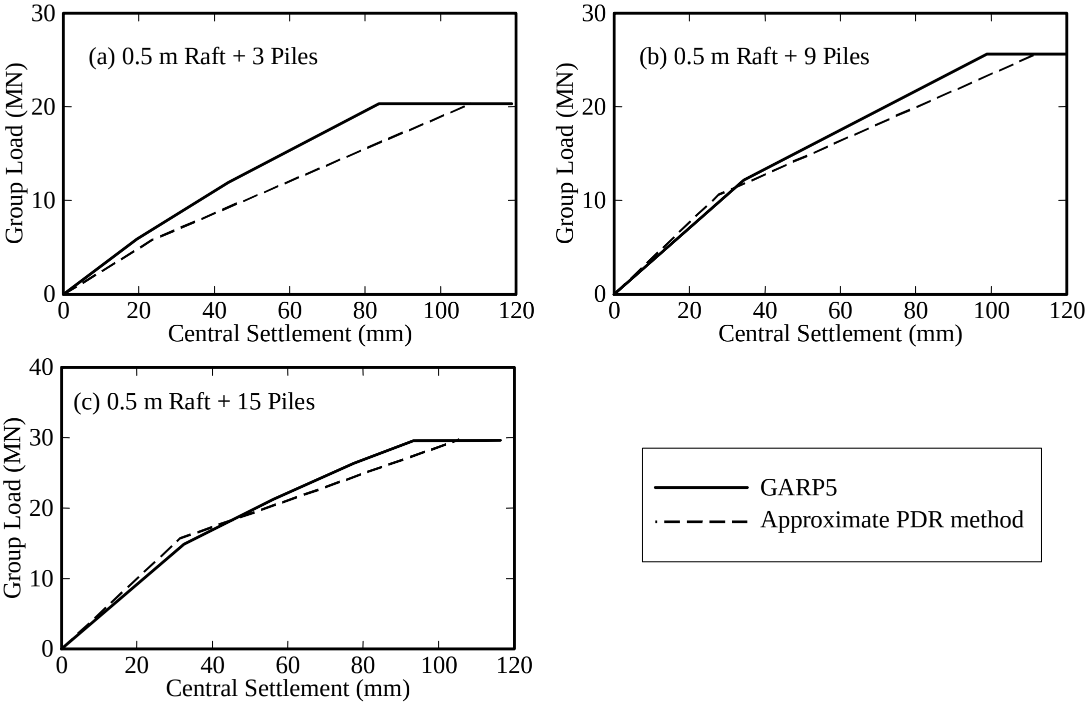
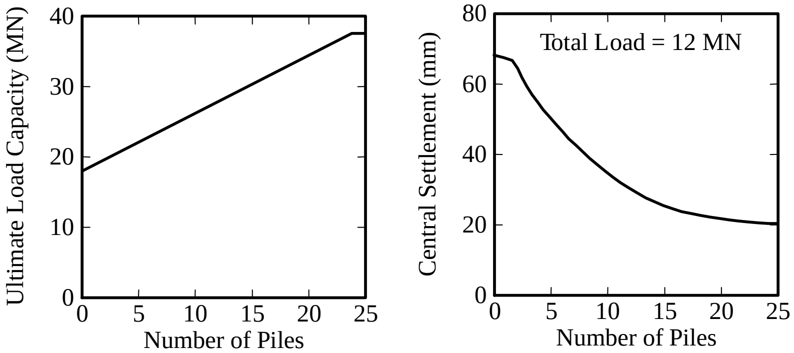


Figure 9. Comparison between GARP and approximate analyses.



(a) Ultimate Load Capacity

(b) Settlement

Figure 10. Effect of number of piles and ultimate load capacity and settlement.

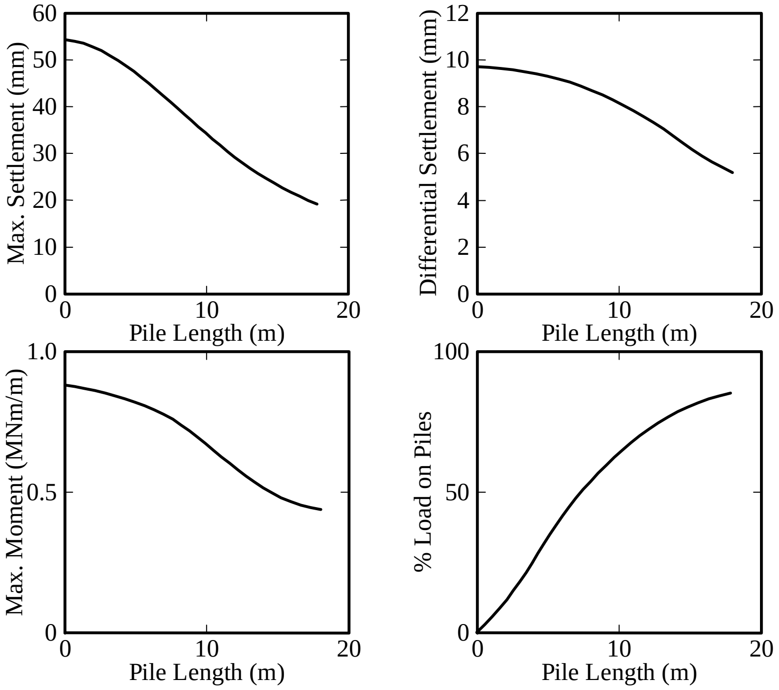


Figure 11. Effect of pile length on foundation performance 0.5m raft with 9 piles, load = 12MN.

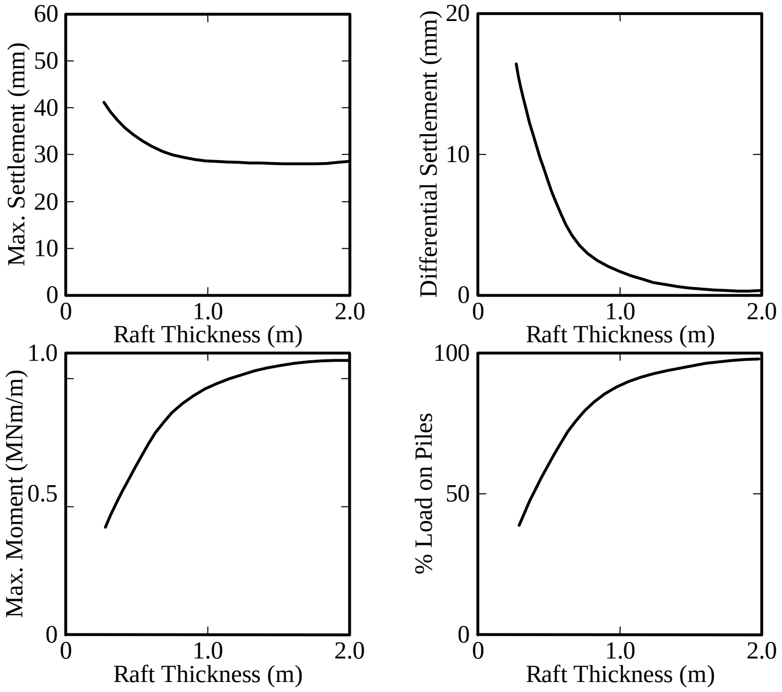


Figure 12. Effect of raft thickness on foundation performance. Raft with 9 piles, 10m long, load = 12MN.

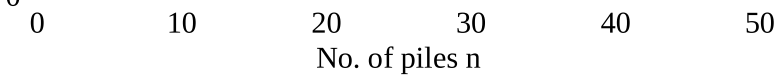


Fig.16 Interaction-diagram: Settlement reduction s/$s_{sf}$ versus L/d and n (Katzenbach et al, 1998)

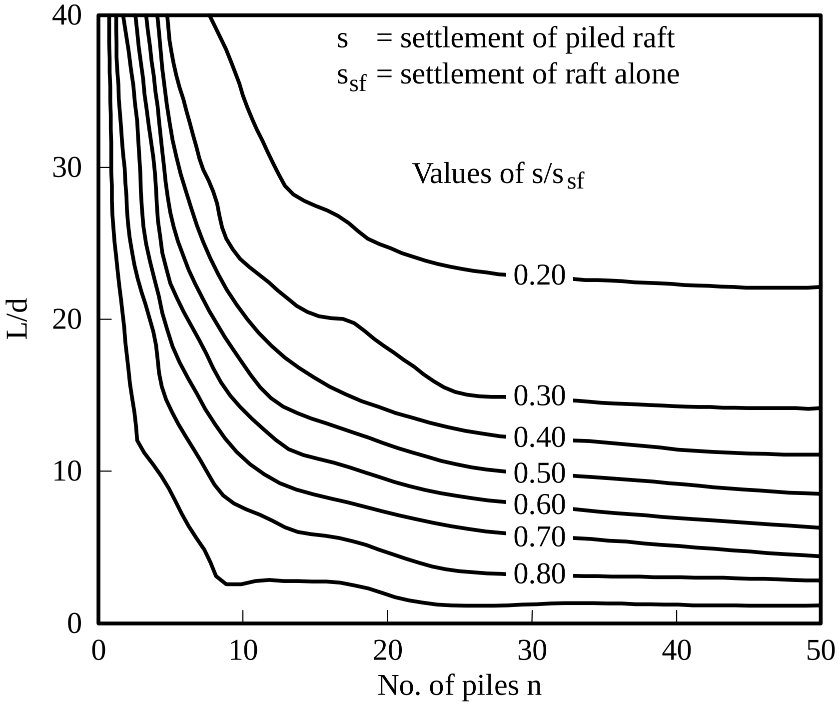


Fig.16 Interaction-diagram: Settlement reduction s/$s_{sf}$ versus L/d and n (Katzenbach et al, 1998)

Figure 13 Interaction diagram : Settlement reduction s/$s_{sf}$ versus L/d and n (Katzenbach et al, 1998)

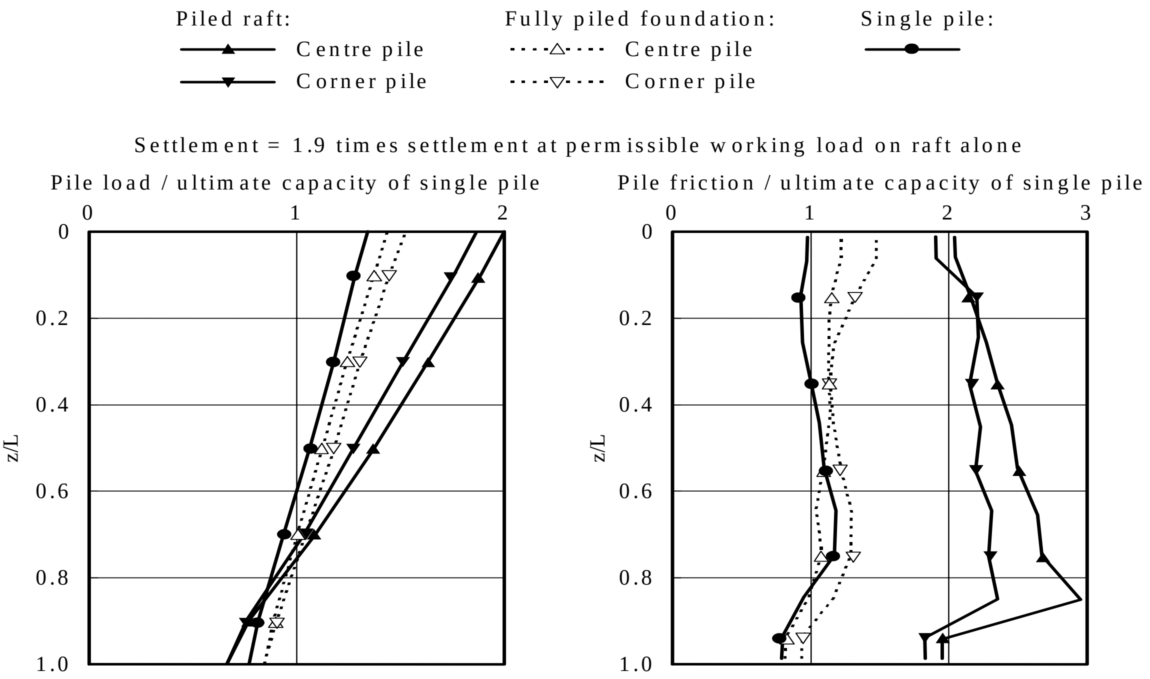


Fig.17 Distribution of the pile load and the skin friction along the pile shaft - raft with 13 piles (Katzenbach et al, 1998)

Figure 14 Distribution of pile load and skin friction along pile shaft – raft with 13 piles (Katzenbach et al, 1998)

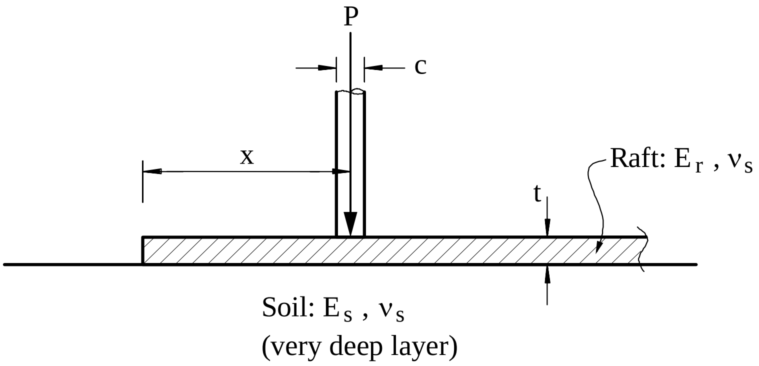


Fig.5 Definition of problem for an individual column load

Figure 15 Definition of problem for an individual column load

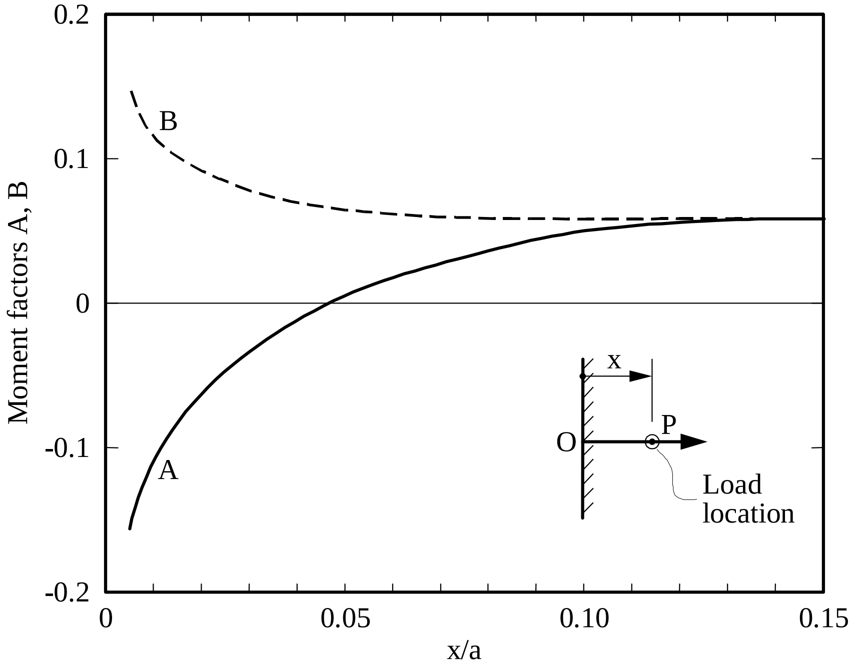


Fig.6 Moment factors A & B for circular column

Figure 16 Moment factors A & B for circular column

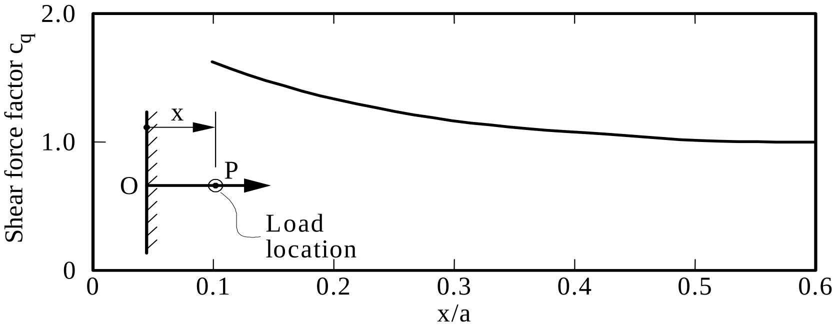


Fig.7 Shear factors $c_q$ for circular column

Figure 17 Shear factors $c_q$ for circular column

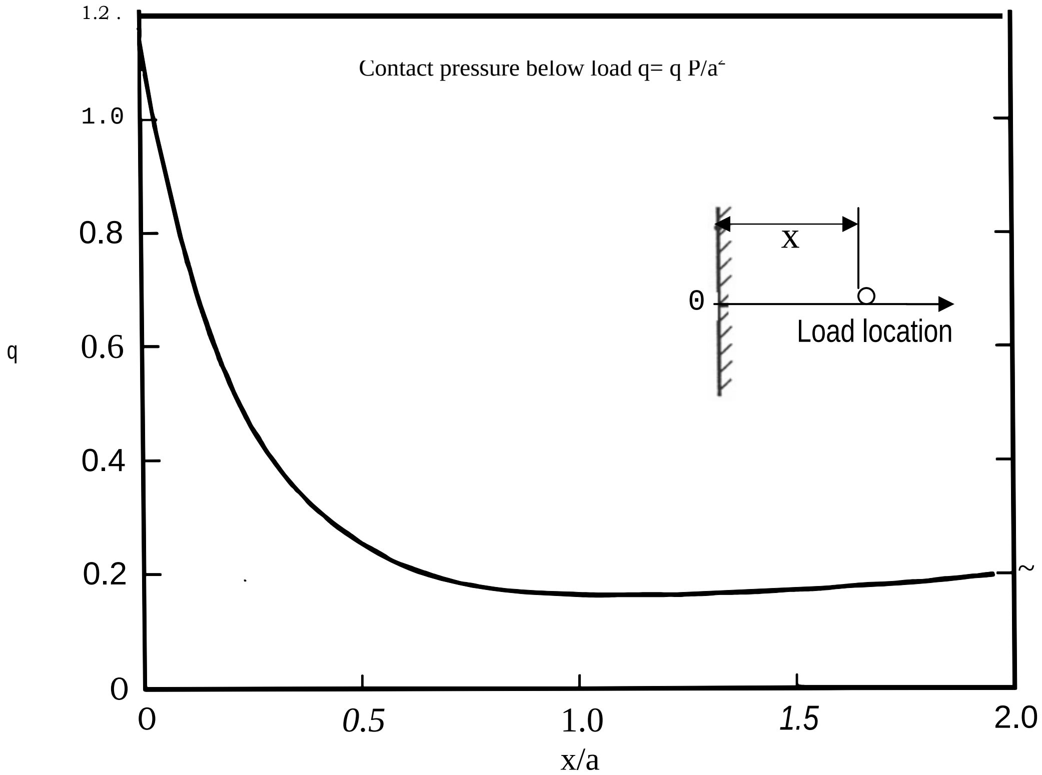


Fig. 18 Contact pressure factor q

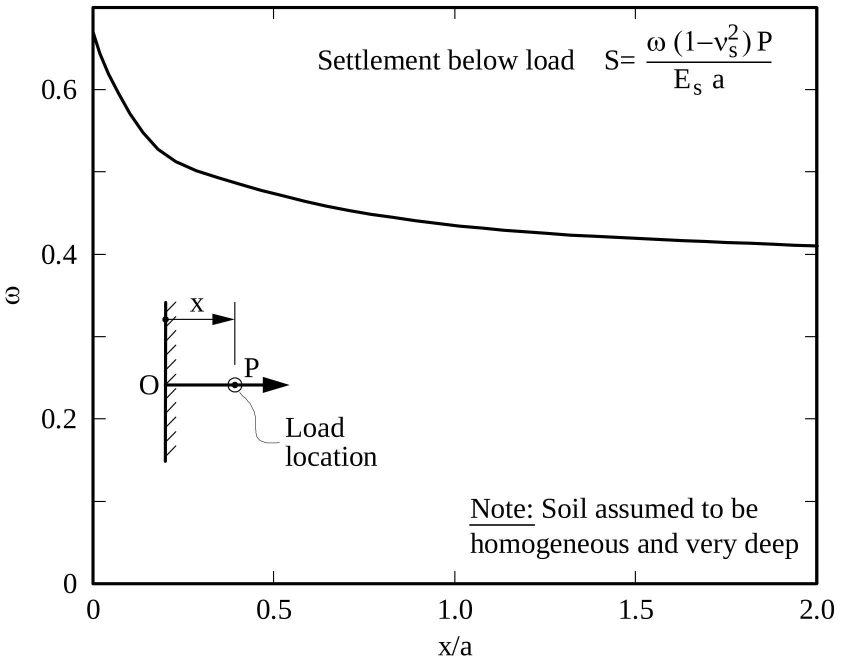
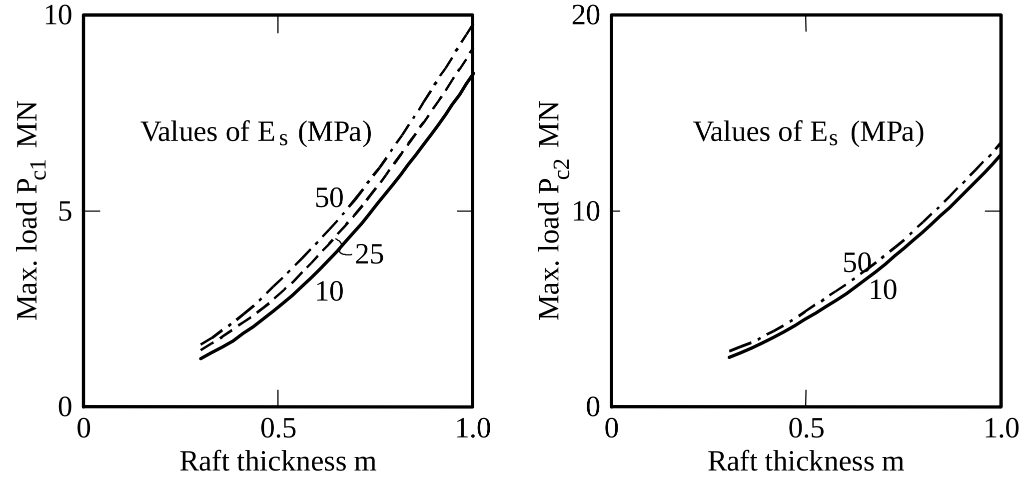


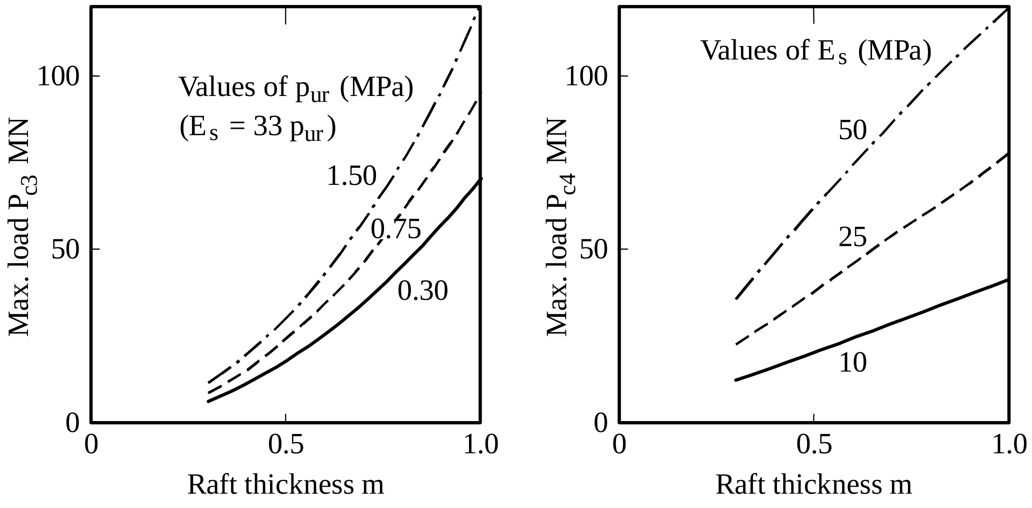
Fig.9 Settlement factor ω

Figure 19 Settlement factor ω

Concrete : $f_c$ = 32MPa $E_r$ = 25000MPa
Steel : $f_y$ = 400MPa 1% reinforcement



(a) Maximum moment criterion (b) Maximum shear criterion



(c) Maximum contract pressure criterion (FS= 1.2)

(b) Maximum local settlement criterion (20mm maximum)

Fig.10 Example of maximum column loads for various criteria - internal columns

Factor 20 Example of maximum column loads for various criteria – internal columns

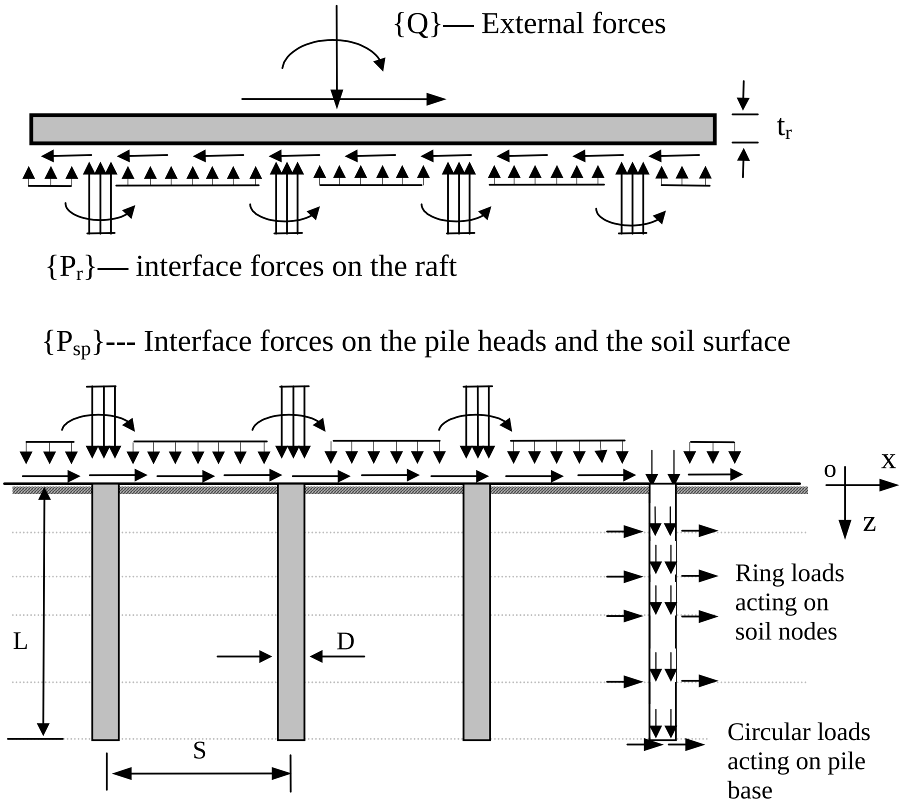


Figure 21. Free body diagram of piled raft with external forces and interface forces in all directions (the y direction is not shown)

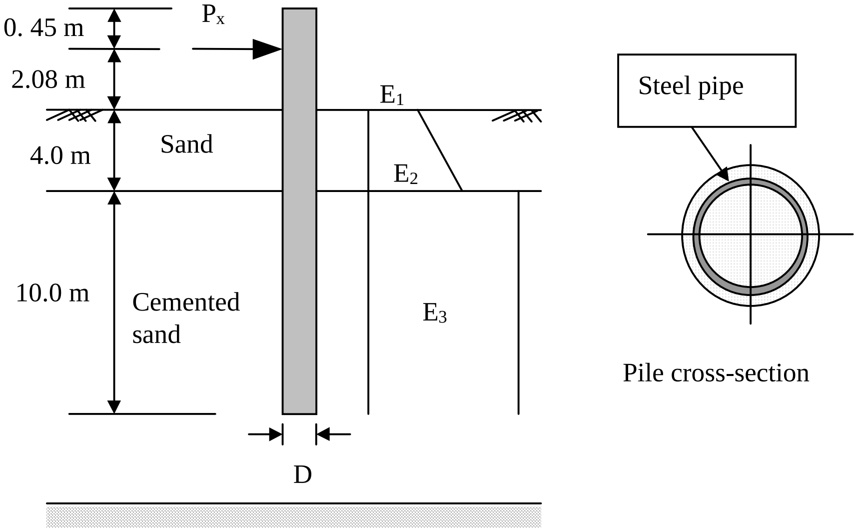


Figure 22. Schematic diagram of single pile and soil profile

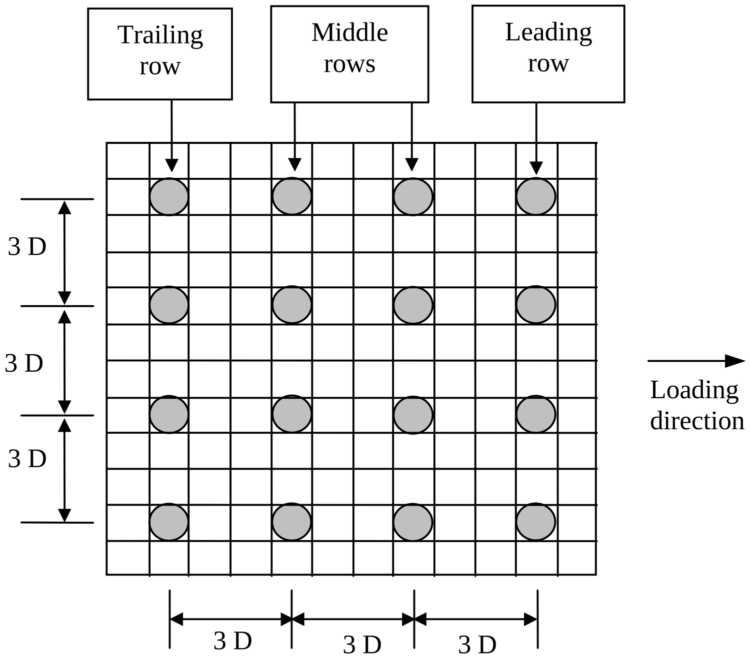


Figure 23. Schematic diagram of pile group

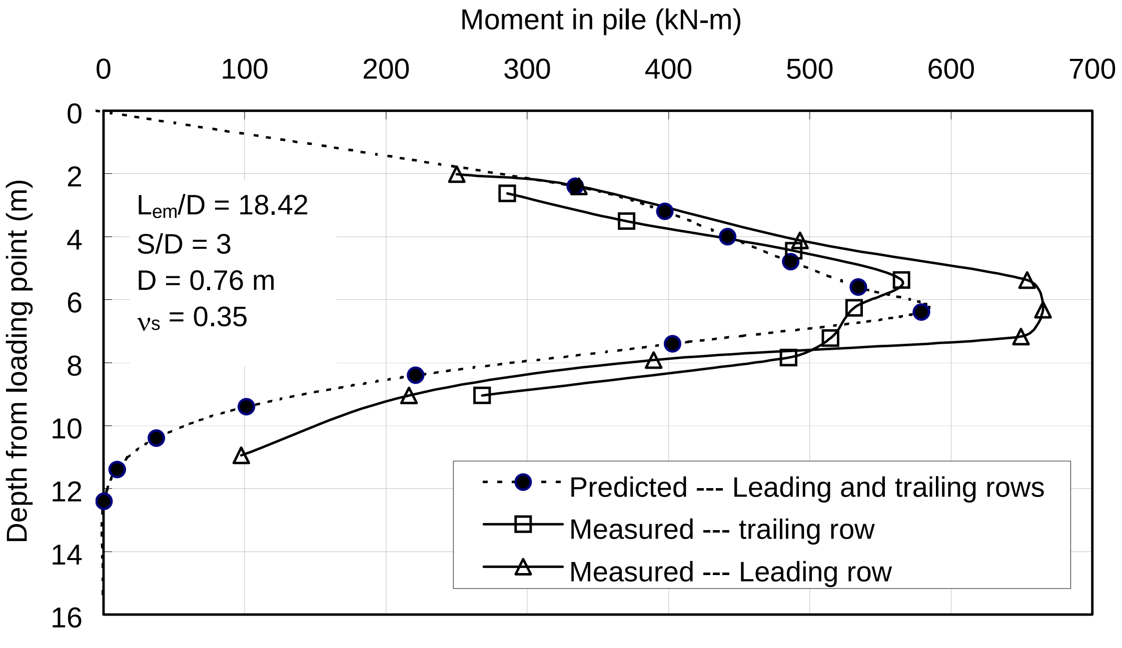


Figure 24. Measured and predicted moment in piles of leading and trailing rows

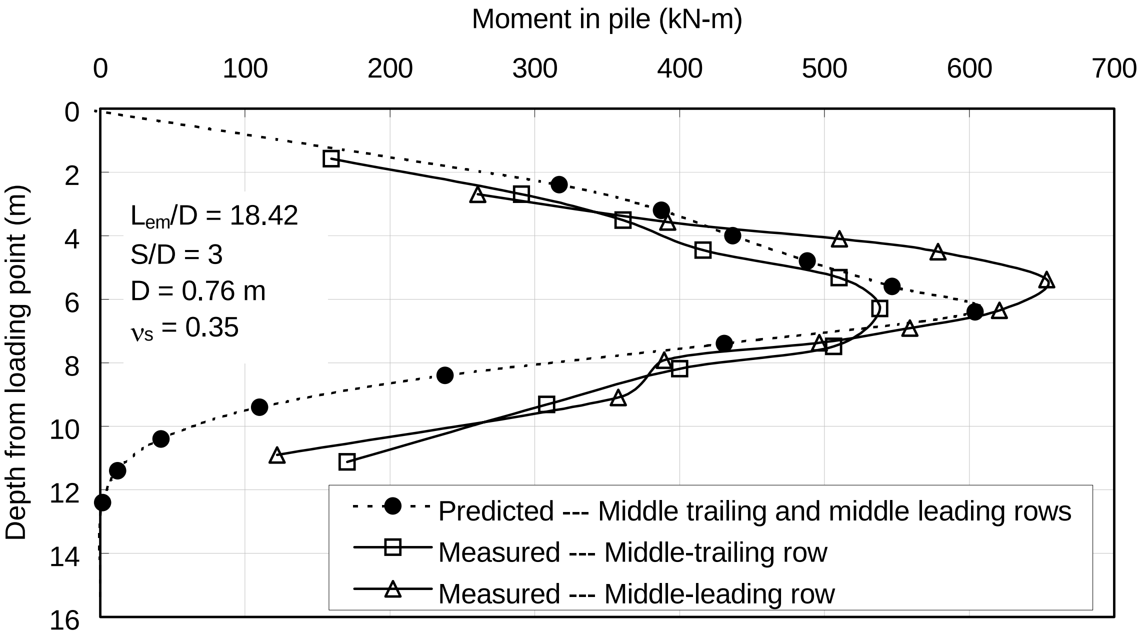


Figure 25. Measured and predicted moment in piles of middle trailing and leading rows

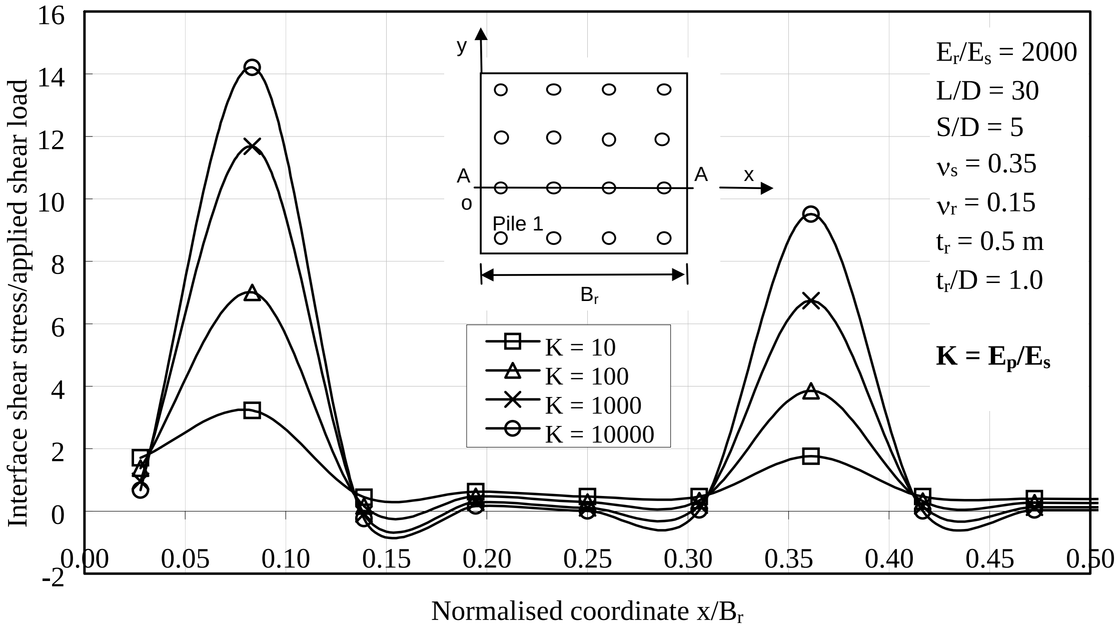


Figure 26. Variation of interface shear pressure along section A-A

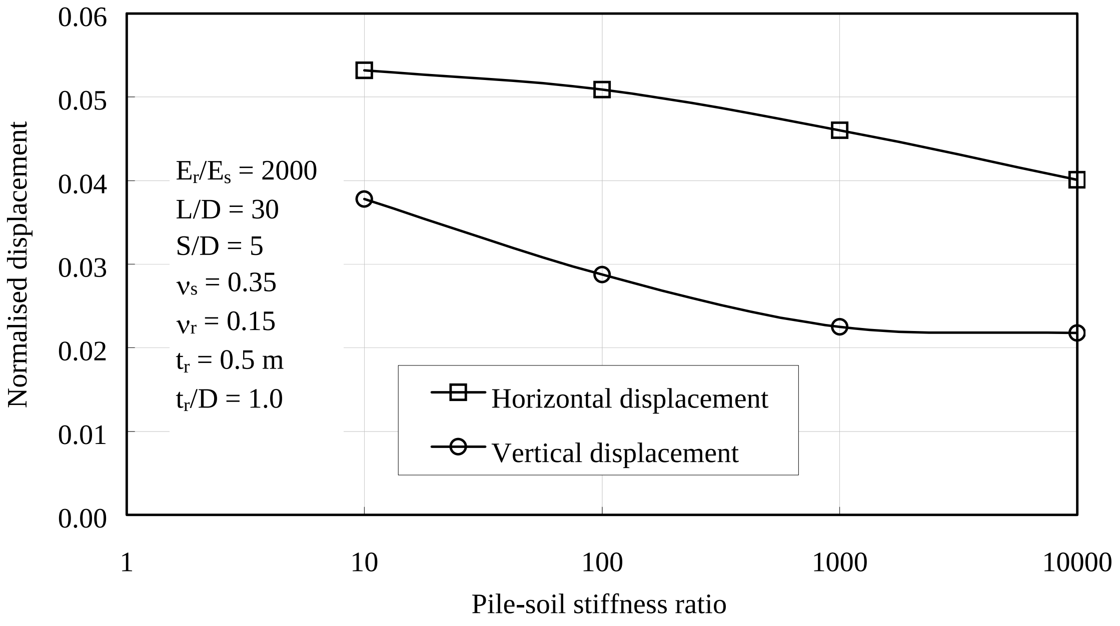


Figure 27. Effect of pile-soil stiffness ratio on displacement of piled raft

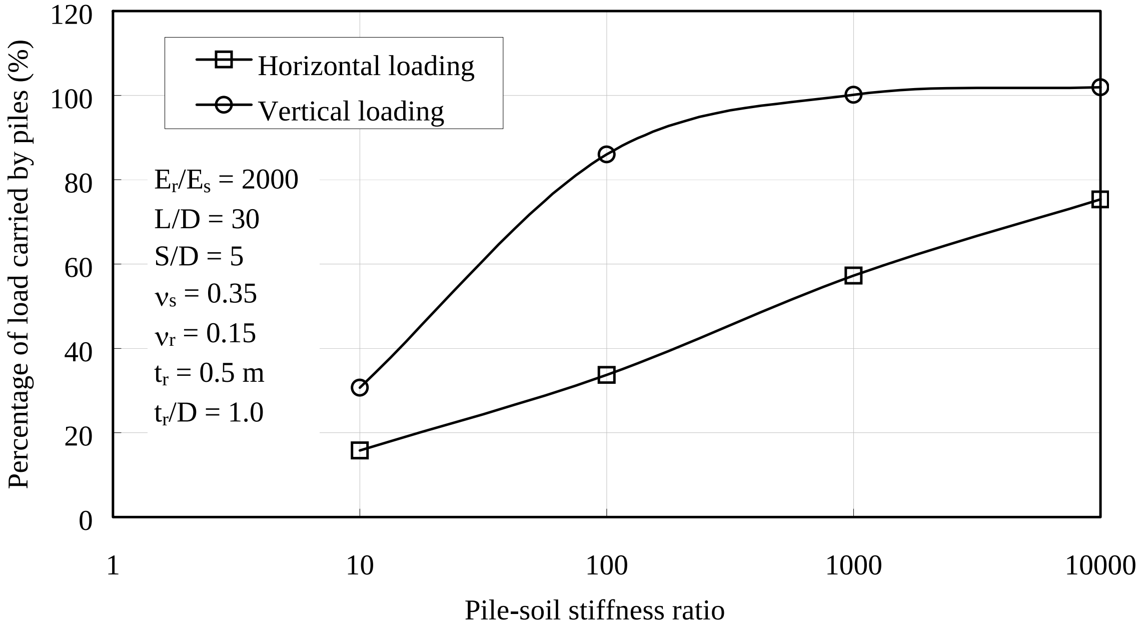


Figure 28. Effect of pile-soil stiffness ratio on load carried by piles

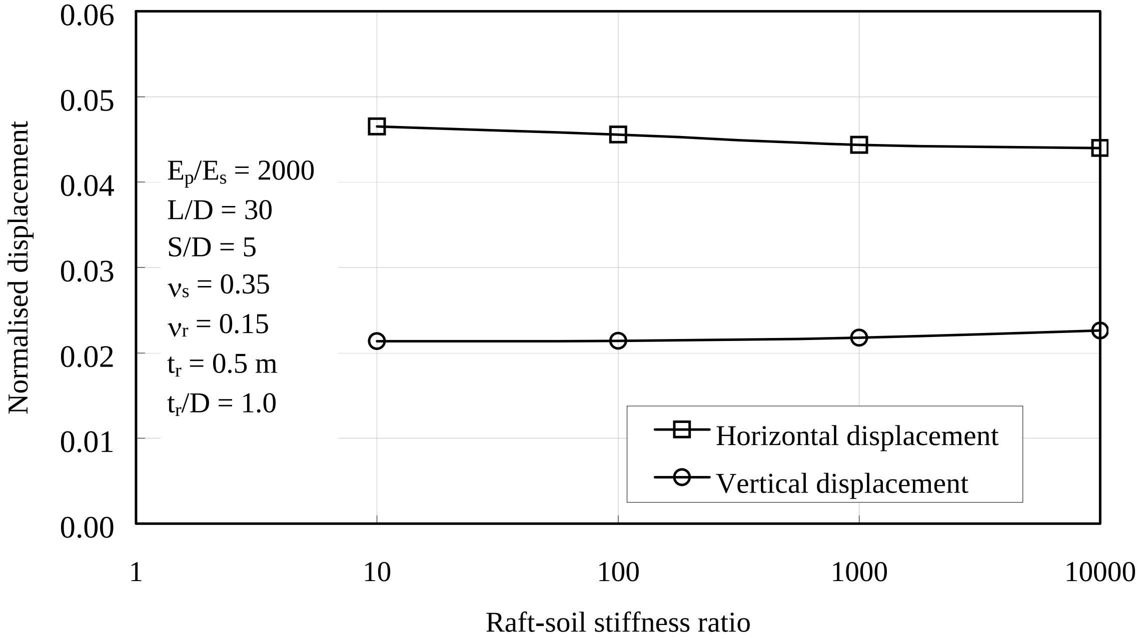


Figure 29. Effect of raft-soil stiffness ratio on displacement of piled raft

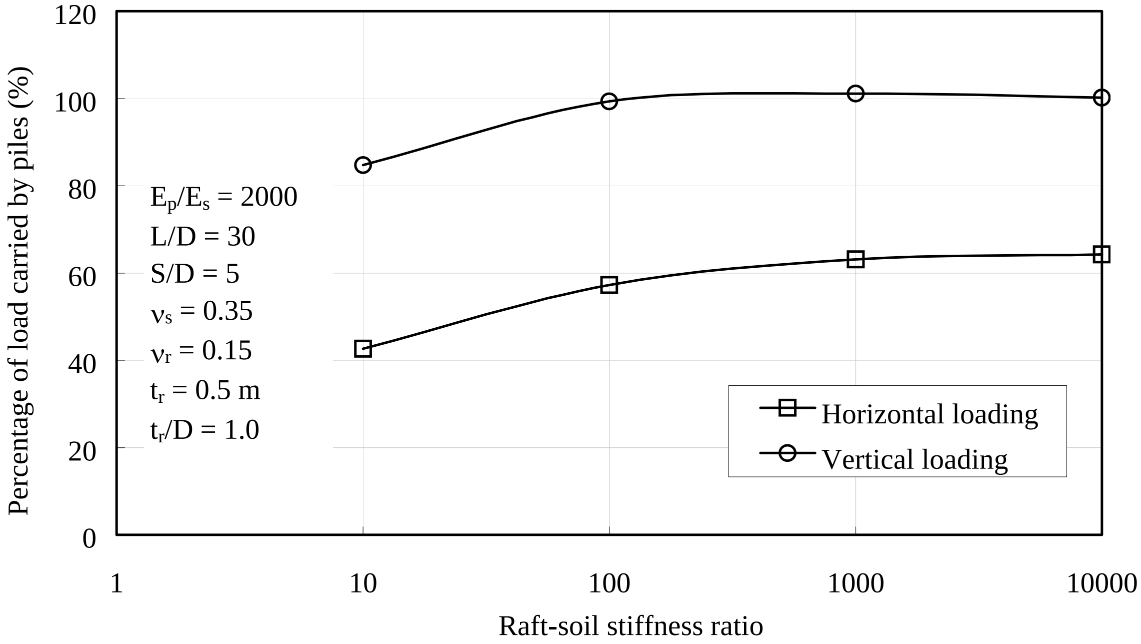


Figure 30. Effect of raft-soil stiffness ratio on load carried by piles

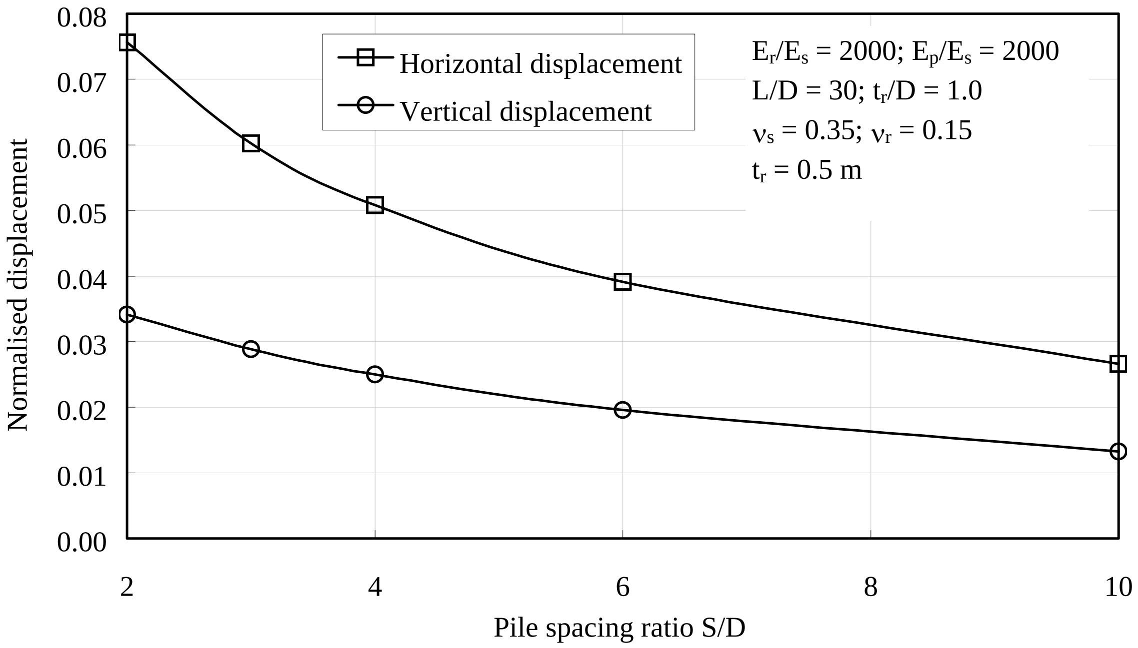


Figure 31. Effect of pile spacing on displacement of piled raft

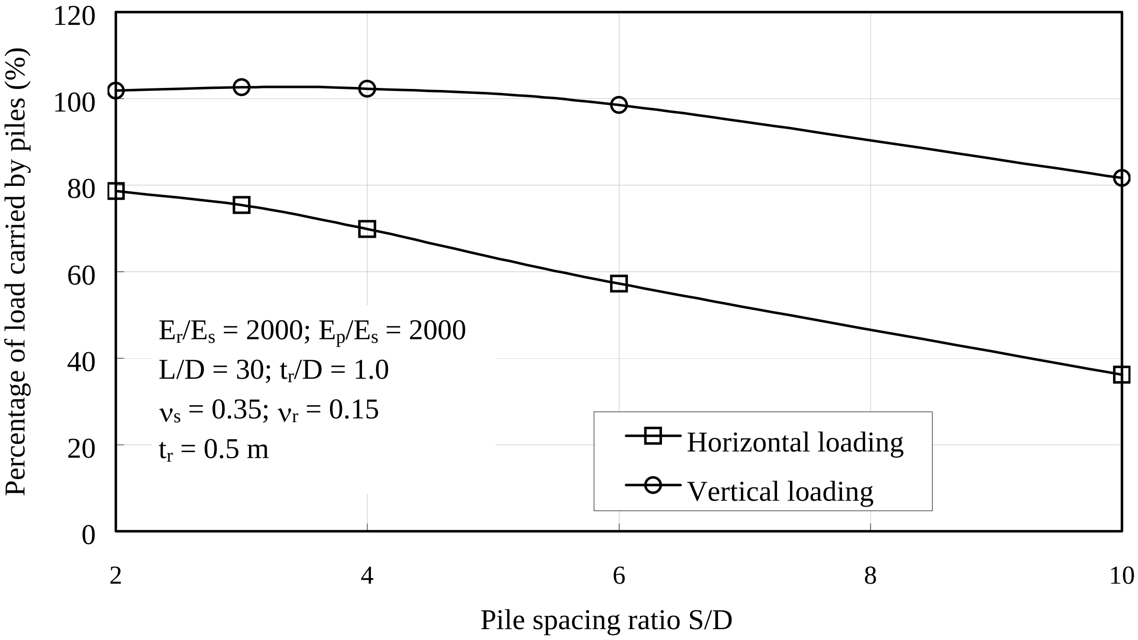


Figure 32. Effect of pile spacing ratio on load carried by piles